

Lunar dynamics and reference systems

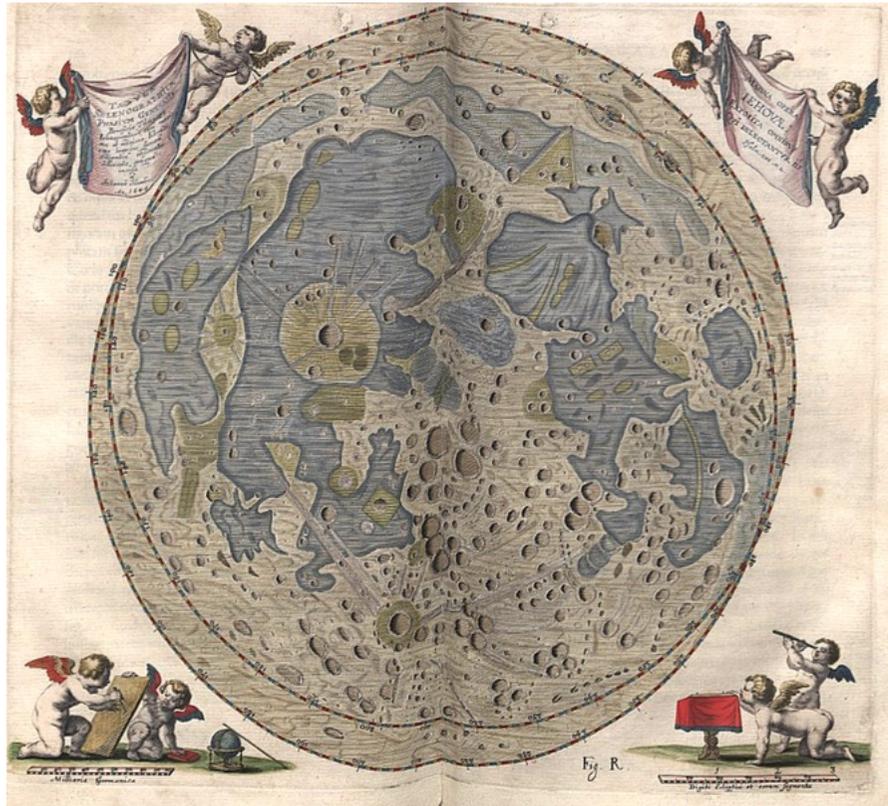
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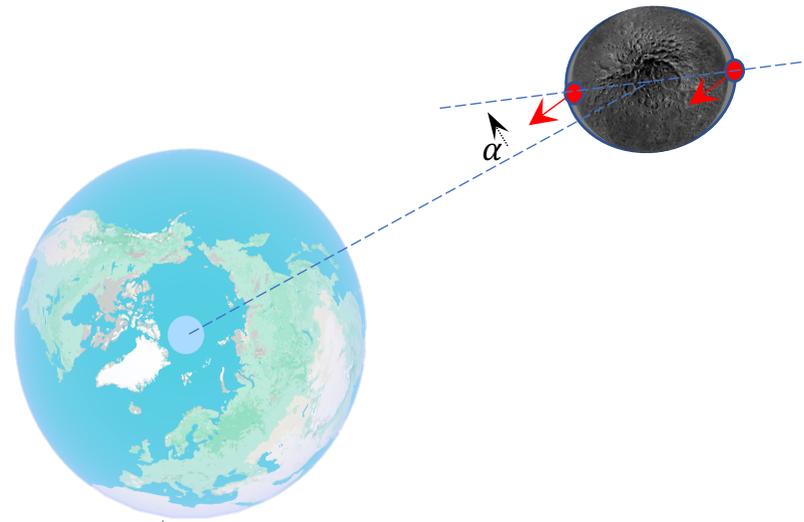
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Lunar libration

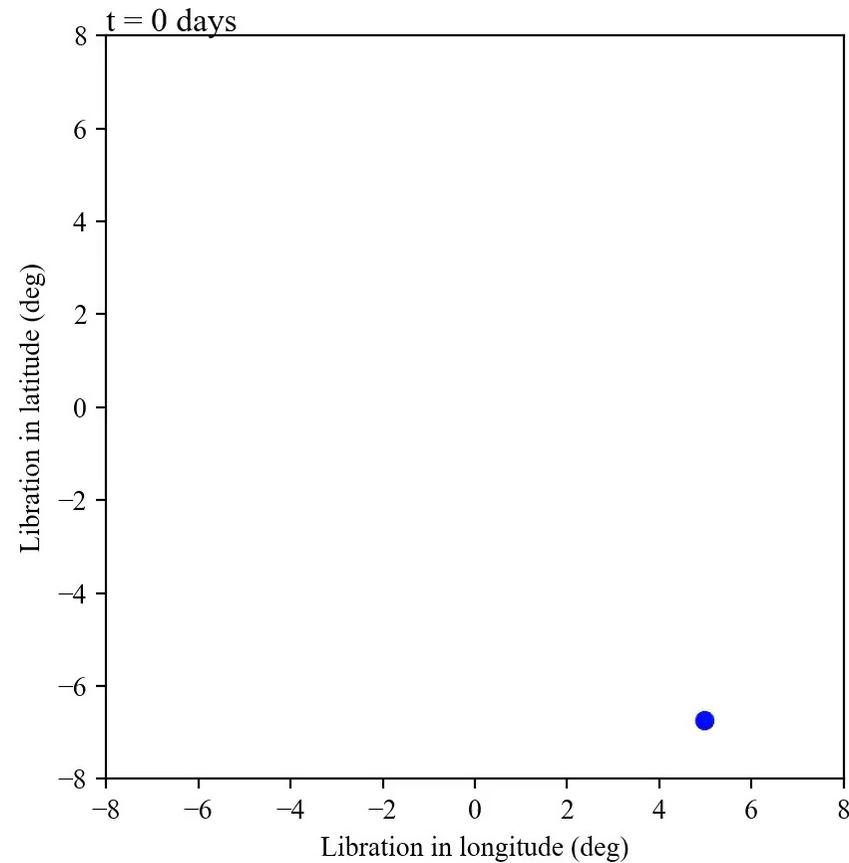


Hevelius 1645
59% of the surface is observable
Optical /geometrical libration



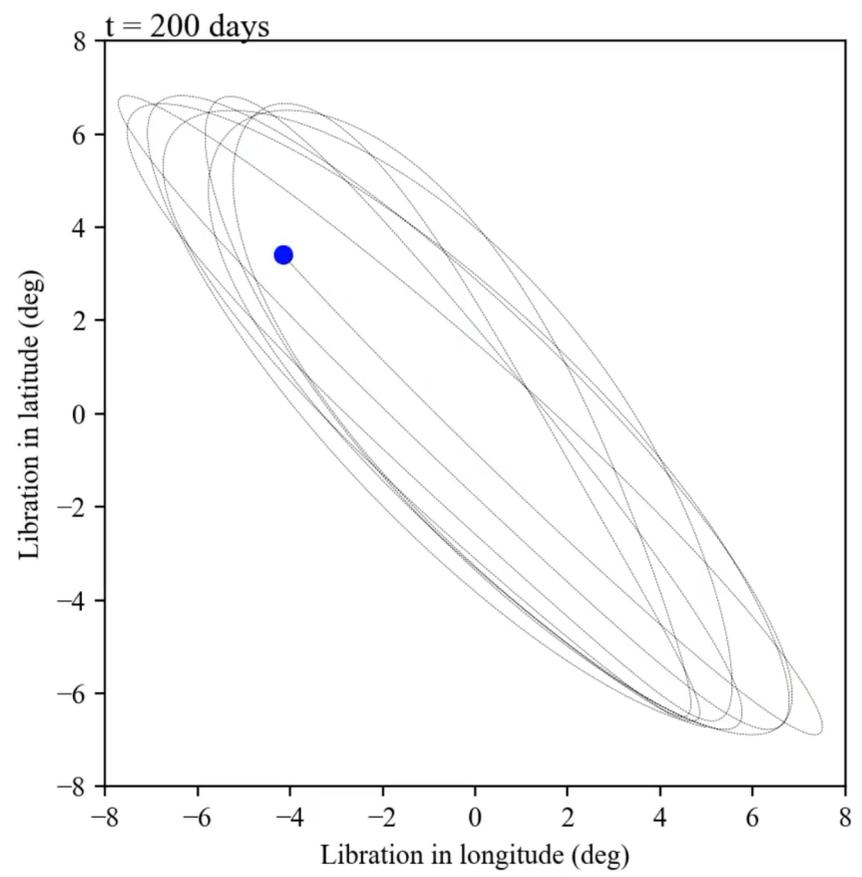
Response to gravitational torque
Physical libration

Earth position seen from the Moon



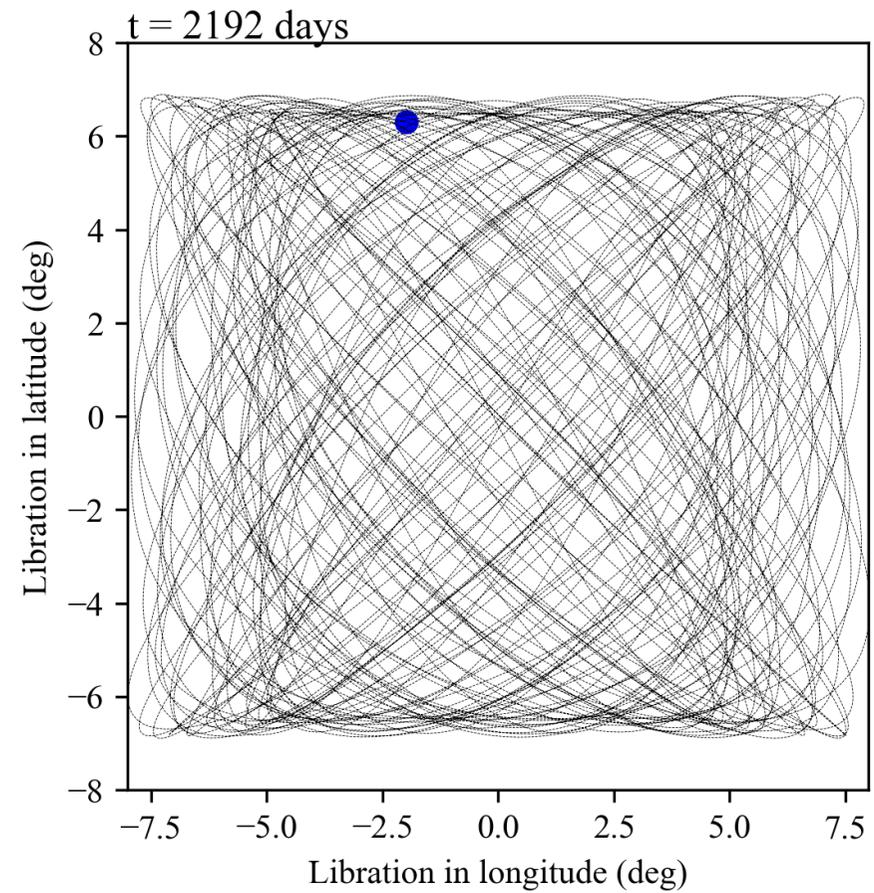
Direction of Earth seen from the PA axis

Earth position seen from the Moon



Direction of Earth seen from the PA axis

Earth position seen from the Moon

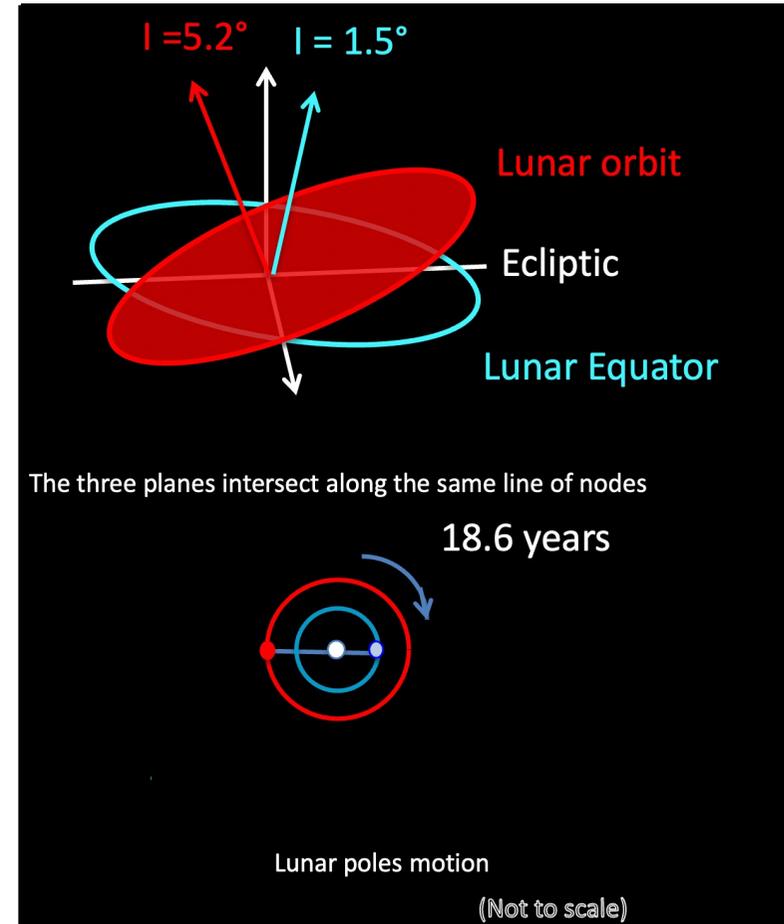


Direction of Earth seen from the PA axis

Cassini Laws

- (1) Sidereal rotation period is 27.32 days, equal to the orbital period;
- (2) The inclination of the Moon's equator to the ecliptic is a small constant angle (1.54°).
- (3) The rotation axis, the orbital normal, and the normal to the ecliptic axis are always in the same plane.

Ref : Cassini 1693; Tisserand 1898; Colombo 1966; Peale 1969; Henrard & Murigande 1987; Bouquillon et al. 2003



$$\theta = I$$

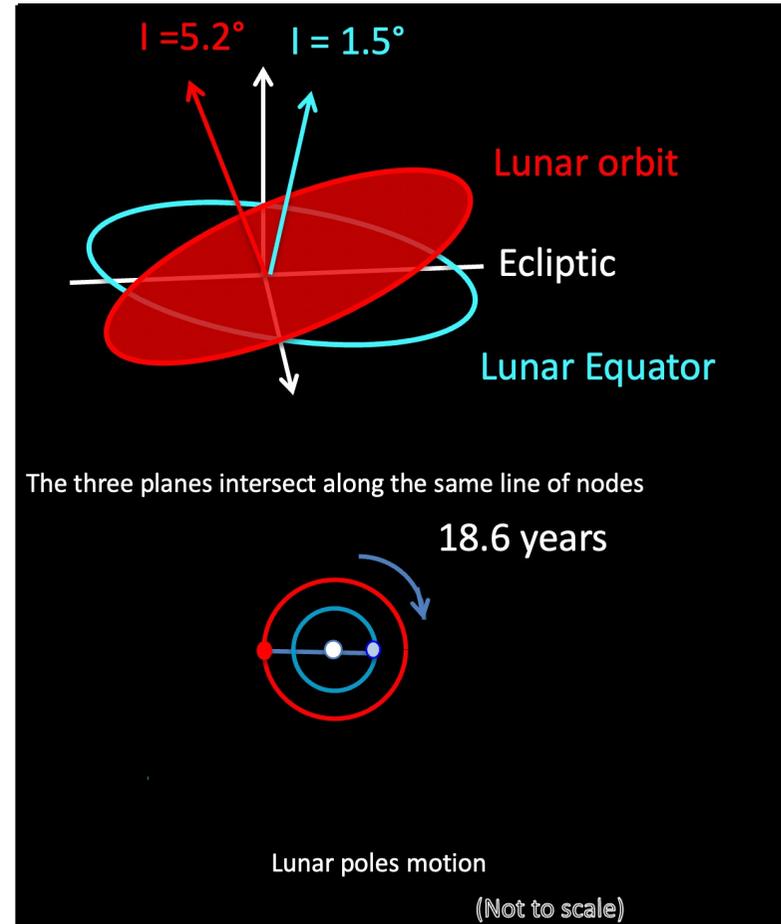
$$\psi = \Omega$$

$$\varphi + \psi = \pi + \lambda$$

Cassini Laws

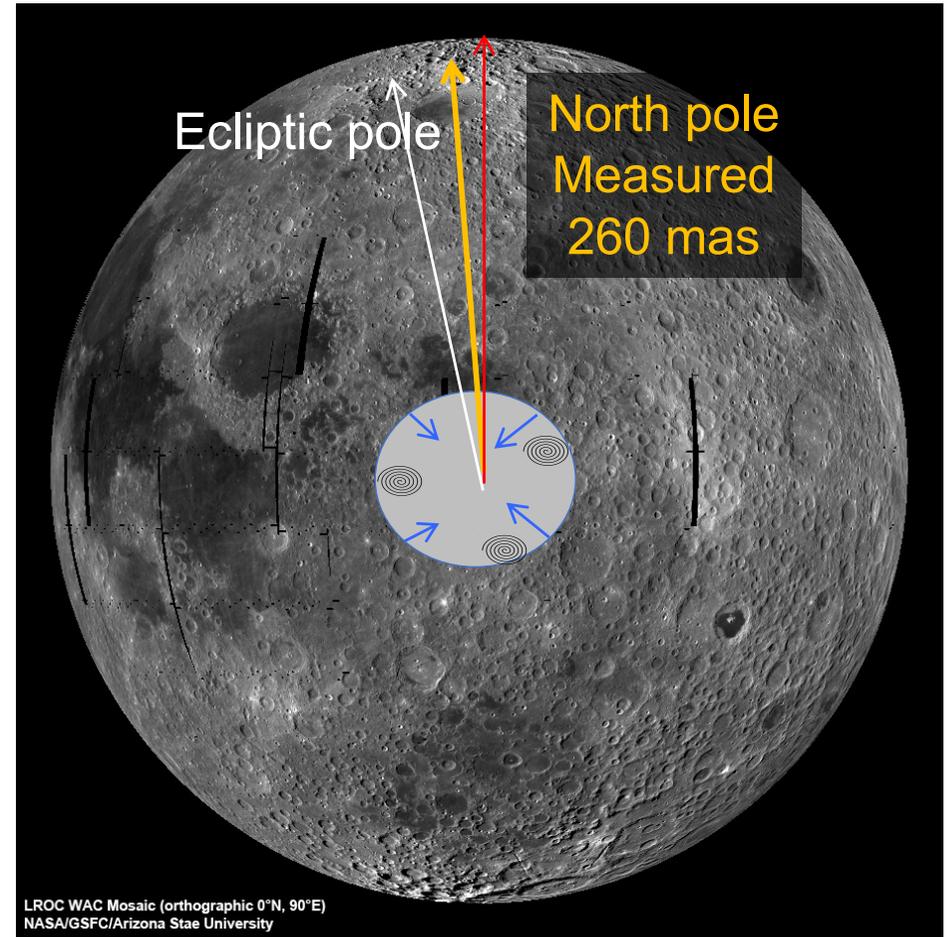
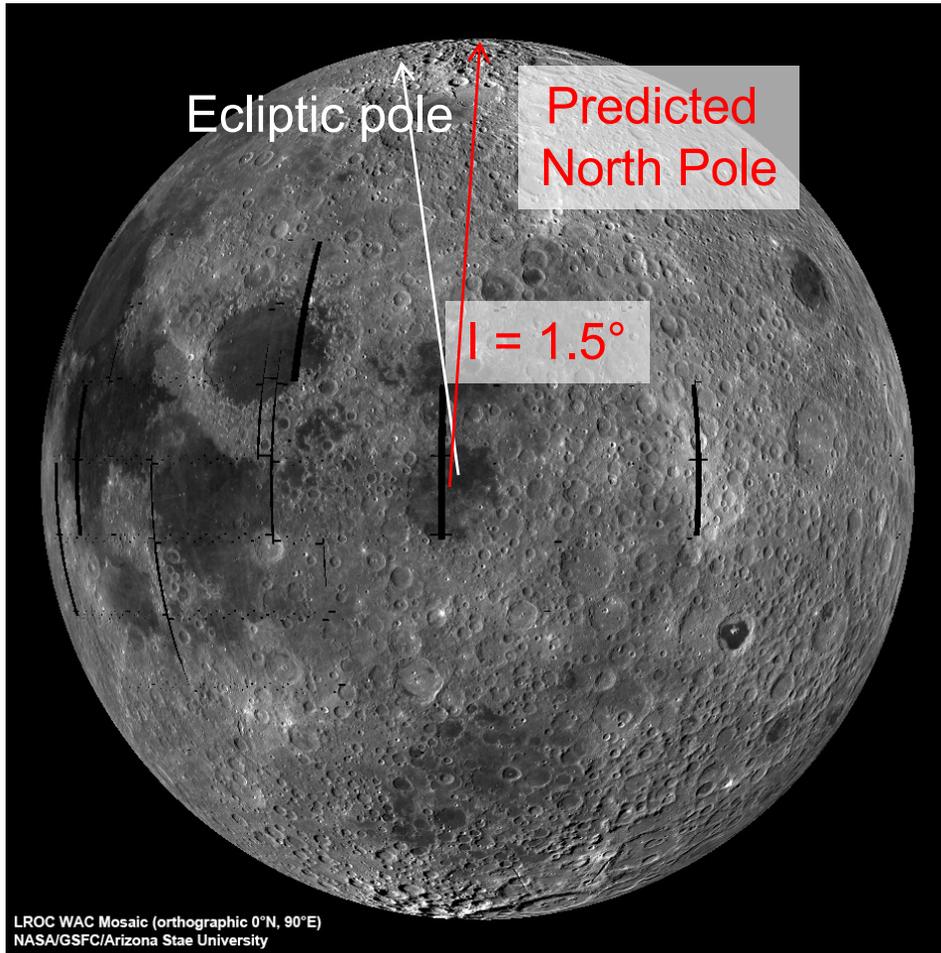
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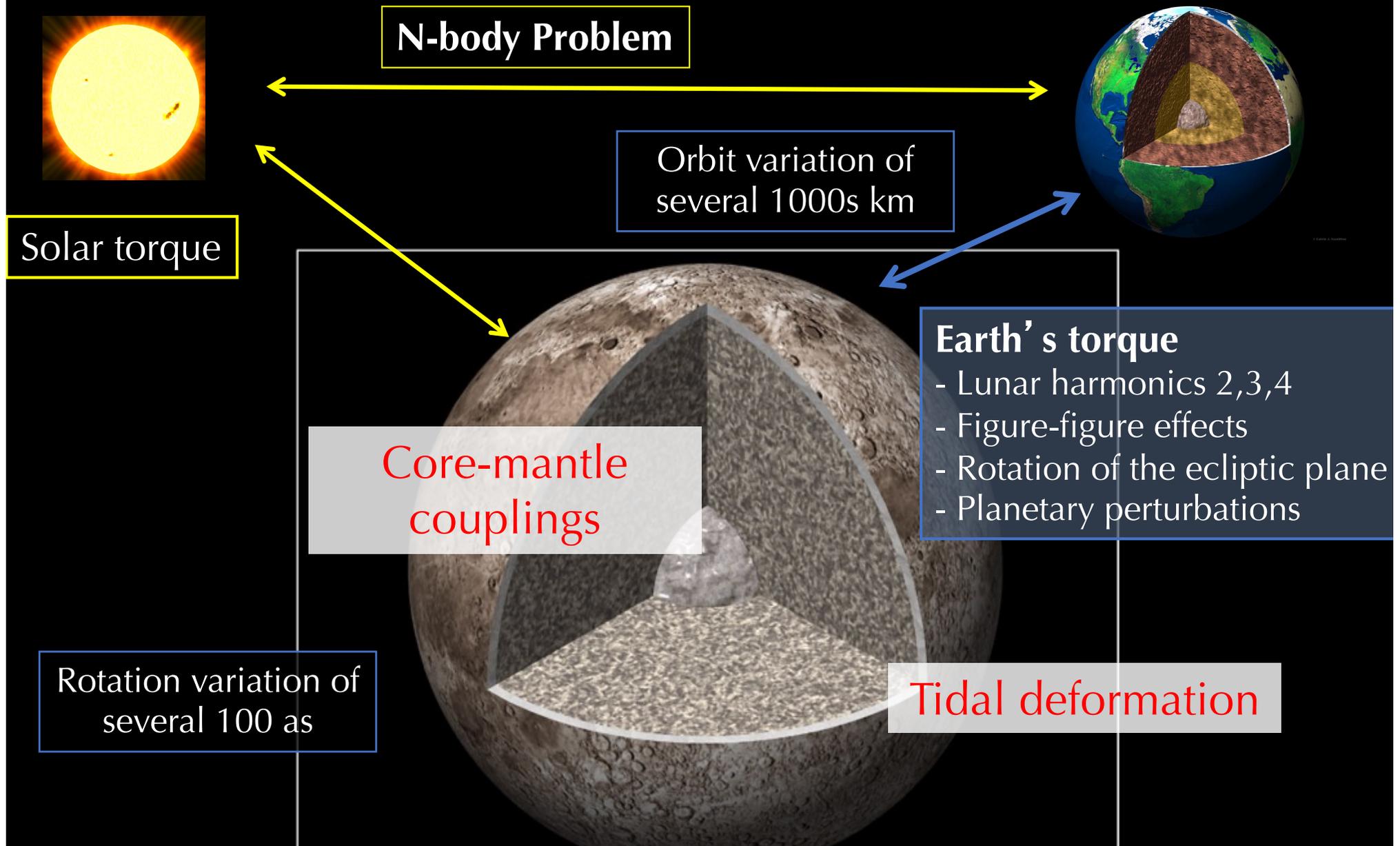
$$\theta = I + \rho \quad \psi = \Omega + \sigma \quad \varphi + \psi = \pi + \lambda + \tau$$

Fluid core



Detection of a fluid core by LLR
(Williams et al. 2001,
Viswanathan, Rambaux, Fienga, Laskar, Gastineau 2019)

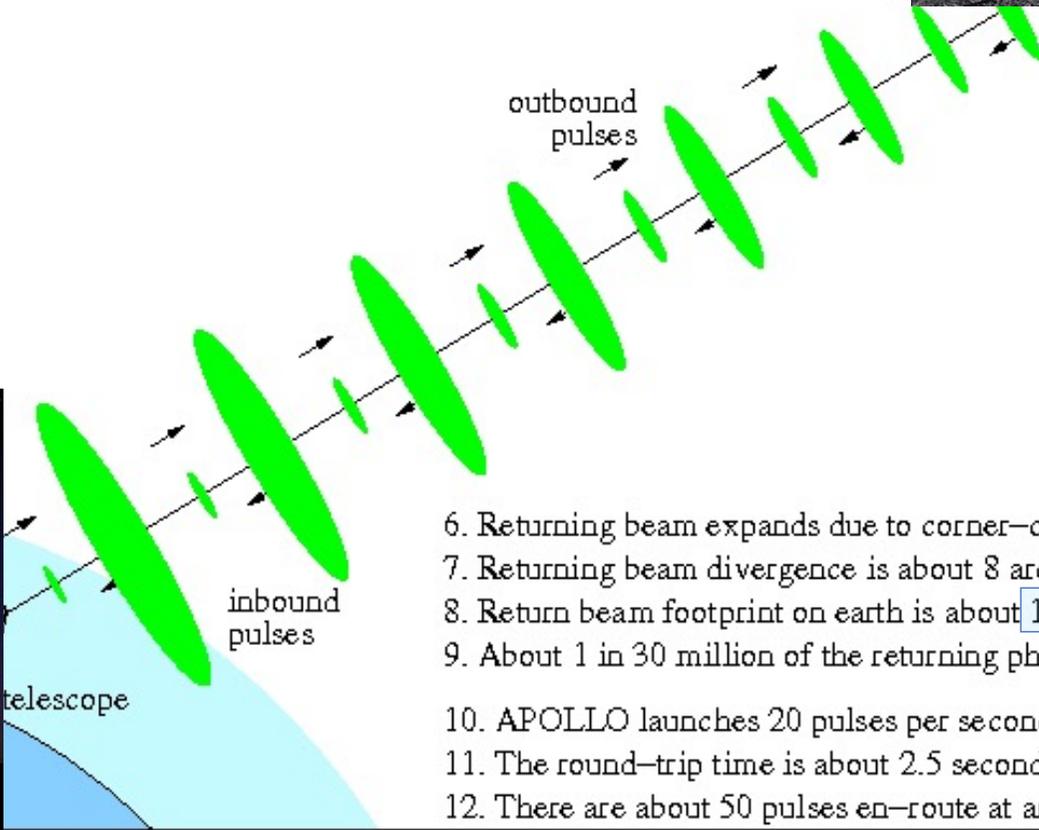
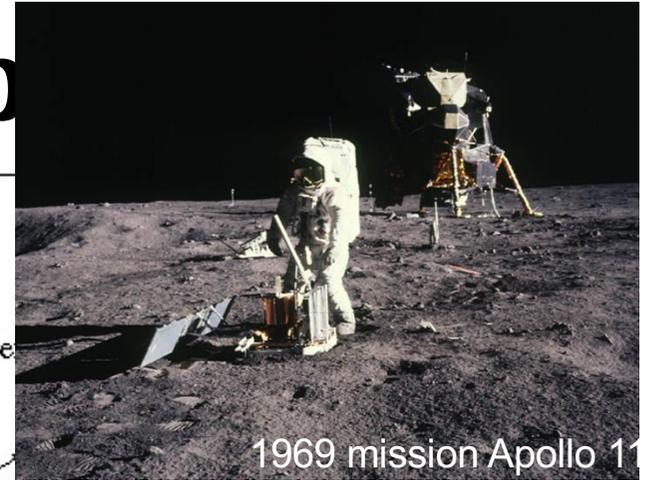
Rotation of the Moon is a complex dynamical system



Librations = departure from a uniform rotational motion

Lunar laser ranging p

1. Outbound pulses start out 3.5 meters in diameter, 2 cm thick
2. Atmosphere causes beam to diverge by one arcsecond or more
3. At the moon, 1 arcsecond is 1.8 km, so beam at moon is about 2 km across
4. Only about 1 in 30 million photons in this 2 km beam hit the suitcase-sized re
5. Each outgoing laser pulse contains 300 quadrillion photons

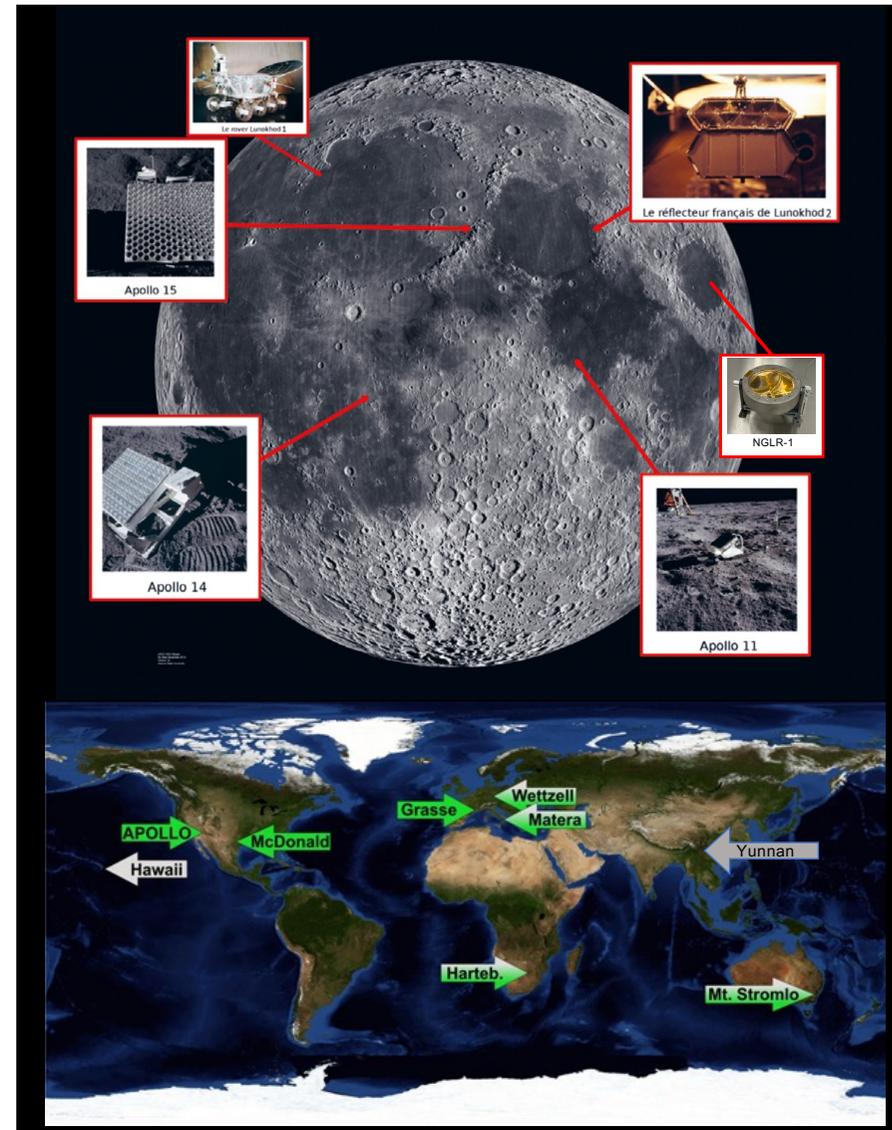


6. Returning beam expands due to corner-cube diffraction
7. Returning beam divergence is about 8 arcseconds
8. Return beam footprint on earth is about 15 km across
9. About 1 in 30 million of the returning photons hit 3.5 m mirror
10. APOLLO launches 20 pulses per second
11. The round-trip time is about 2.5 seconds
12. There are about 50 pulses en-route at any moment in time

Station MEO,
Grasse, France

LLR Measurements

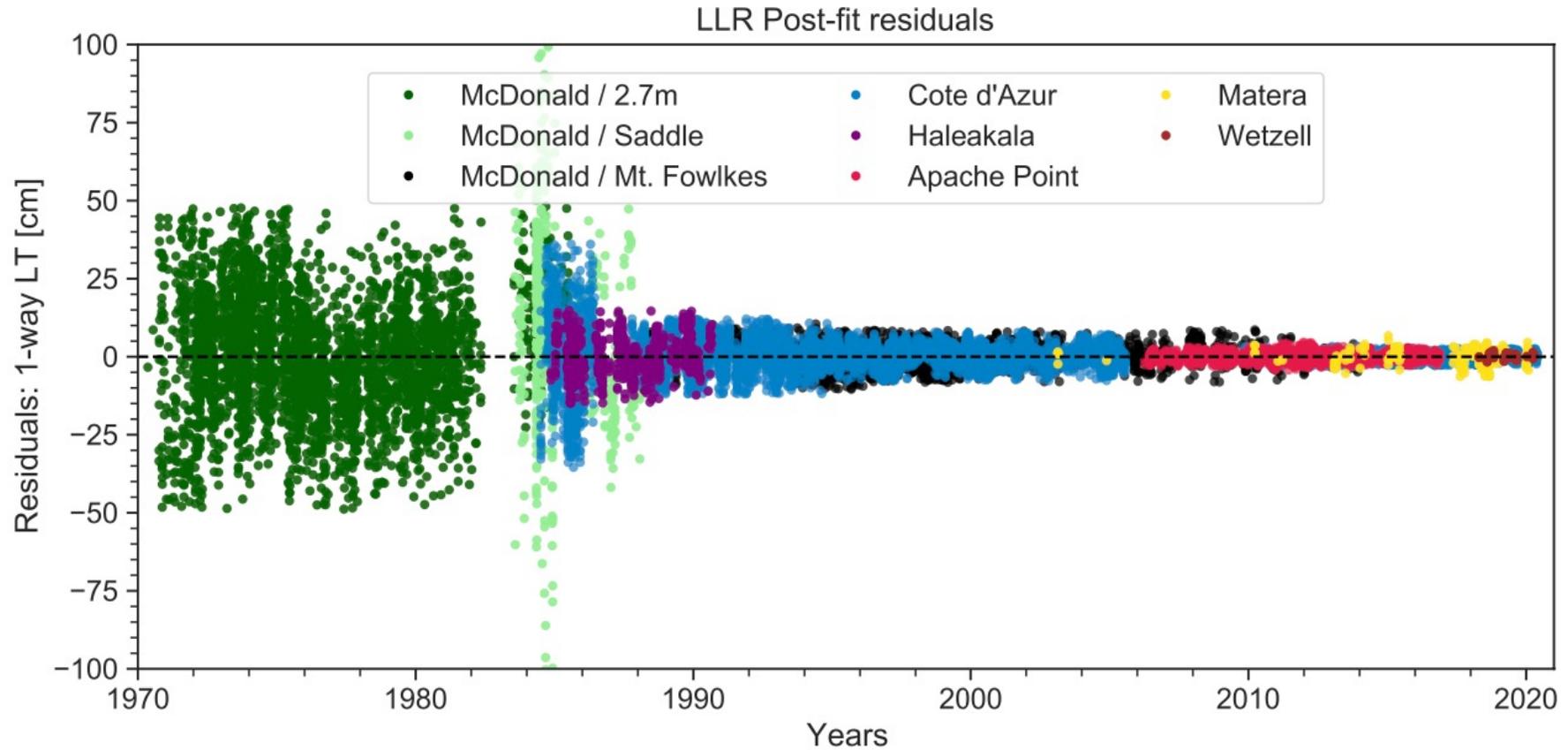
- Time span : 1969-today
- Number of NP : +28000
- Apollo station telescope of 3.5 meters
- Grasse-OCA laser : green and IR
- Earth-Moon distance accuracy of ~ cm
- Theoretical accuracy: few mm



Lunar-Laser Ranging Experiment and ephemerides

- Numerical planetary and lunar ephemerides DE, EPM, IfE and INPOP (e.g. Williams *et al.*, Pavlov *et al.*, Hoffman *et al.* Fienga *et al.*)
 - Lunar accuracy ~ **1 cm** and **1 mas** in rotation over **50 years**.
 - Fundamental physics, geophysics, selenophysics and **interior** of the Moon.
- These models (DE, EPM, INPOP) are joint **numerical** integration of the orbits of the Moon, the Earth, the planets and asteroids, and of the lunar rotation
- Dynamical partial derivatives of the orbits and lunar Euler angles with respect to solution parameters such as **moment of inertia**, gravity field, **tides**, **dissipation**, **CMB flattening**, and initial conditions.

Residuals of the observations



INPOP21a
(Fienga+ 2021)

Construction steps for the lunar reference system/frame

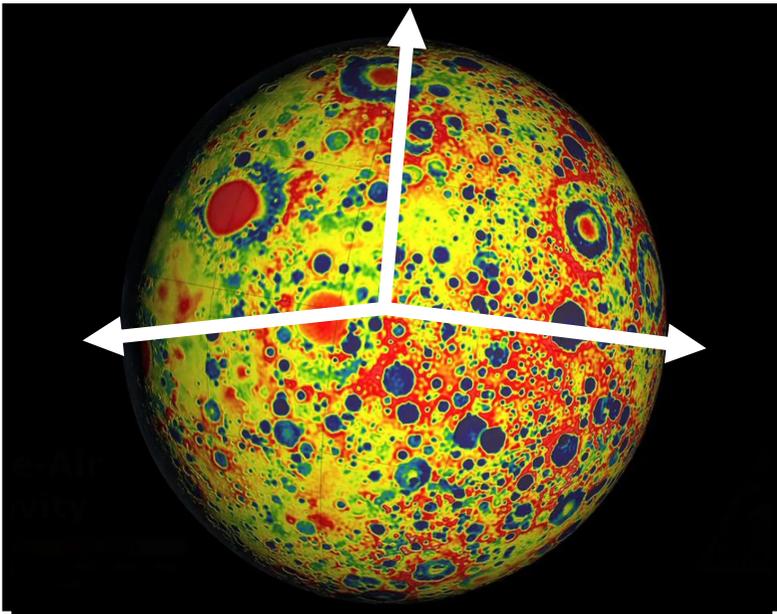
Steps	Specific name	PA	ME
1. Conception	Ideal reference system		
2. Choice of a structure	Reference system		
3. Structure modelling	Conventional reference system		
4. Materialisation	Conventional reference frame		
5. Extension and densification	Secondary reference frame		

(Kovalevsky and Muller 1981)

1. The conception

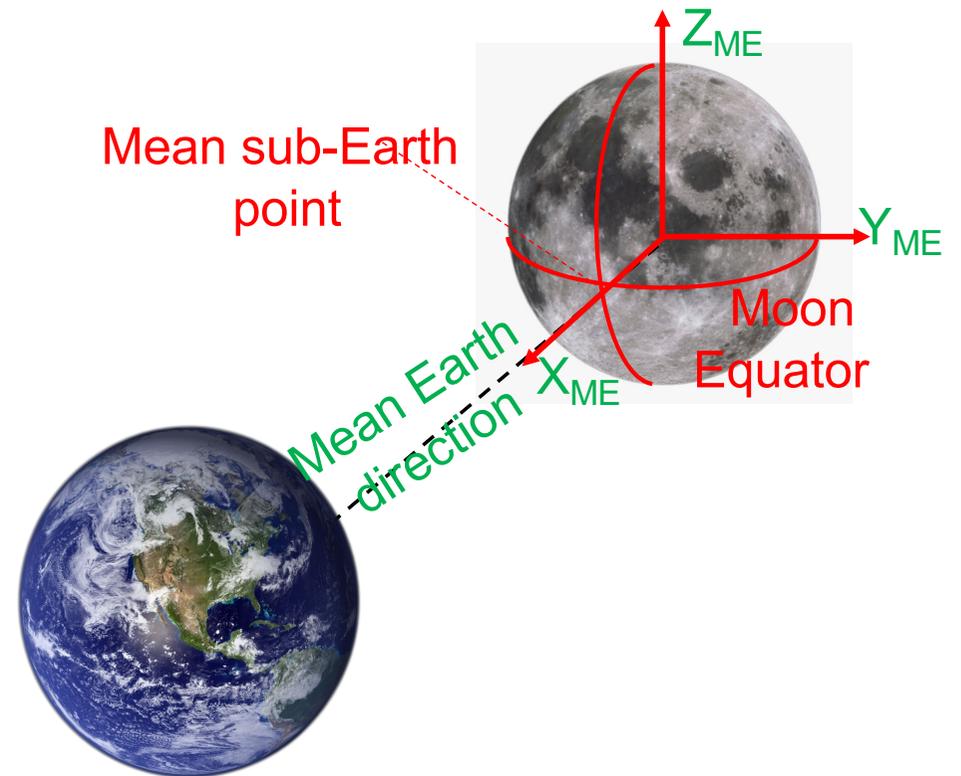
Principal Axis System

(credit NASA/JPL-Caltech/MIT/GSFC)



Diagonal matrix inertia

Mean Earth / Rotation axis System

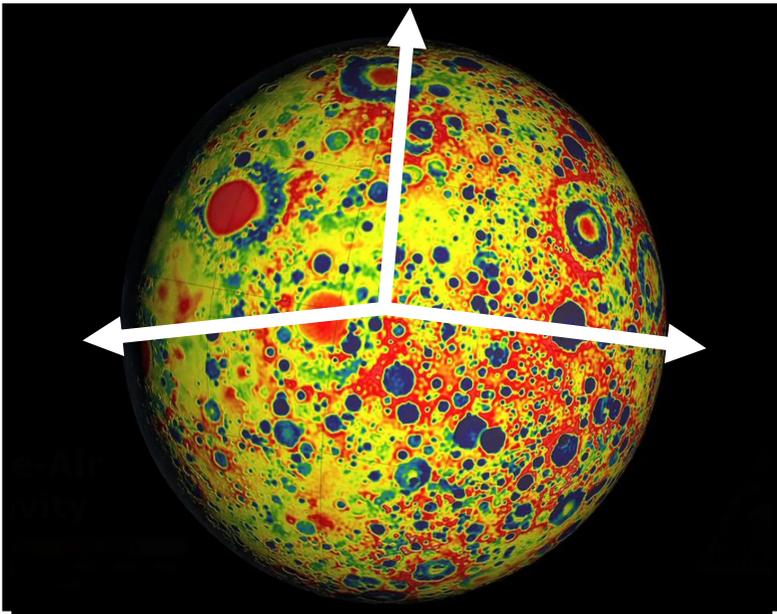


Geometric
Mean Earth direction

2. The choice of a structure

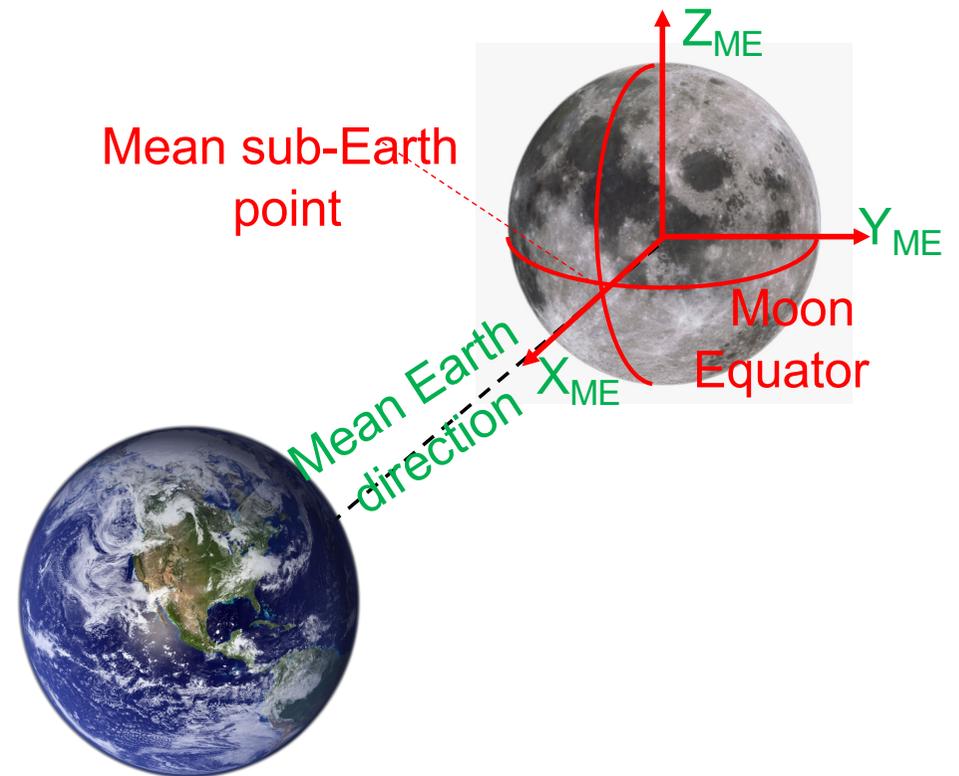
Principal Axis System

(credit NASA/JPL-Caltech/MIT/GSFC)



Diagonal matrix inertia
(undistorted moon)

Mean Earth / Rotation axis System

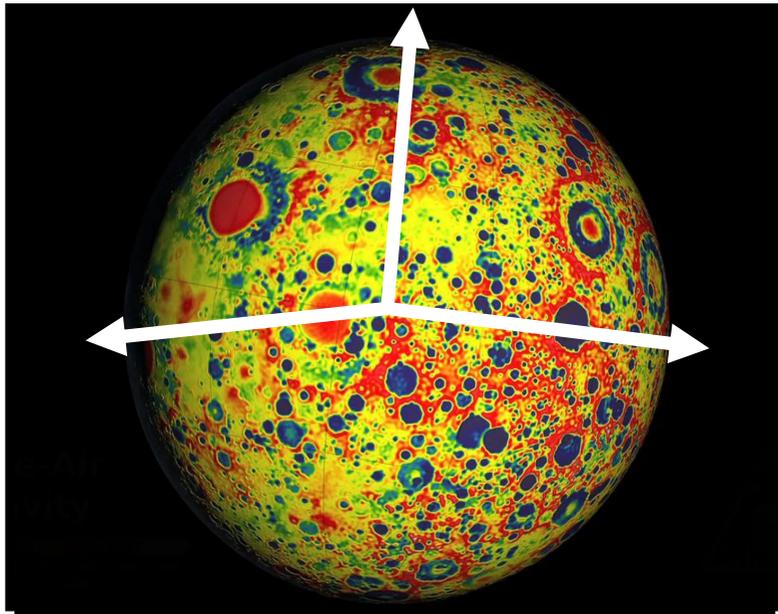


Mean Earth direction
(gravity field, dissipation)

3. Structure modelling

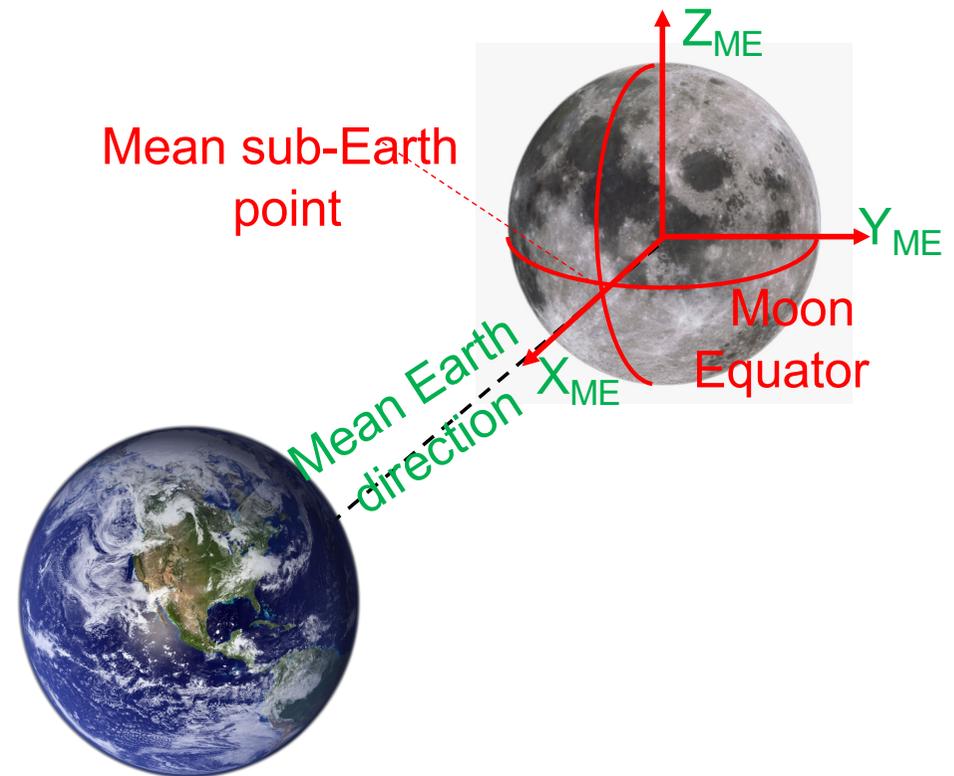
Principal Axis System

(credit NASA/JPL-Caltech/MIT/GSFC)



Diagonal matrix inertia
(undistorted moon)

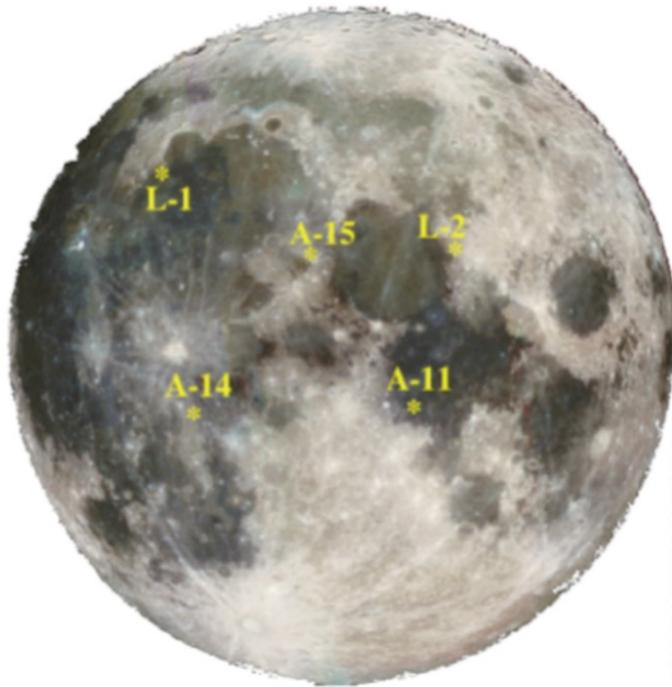
Mean Earth / Rotation axis System



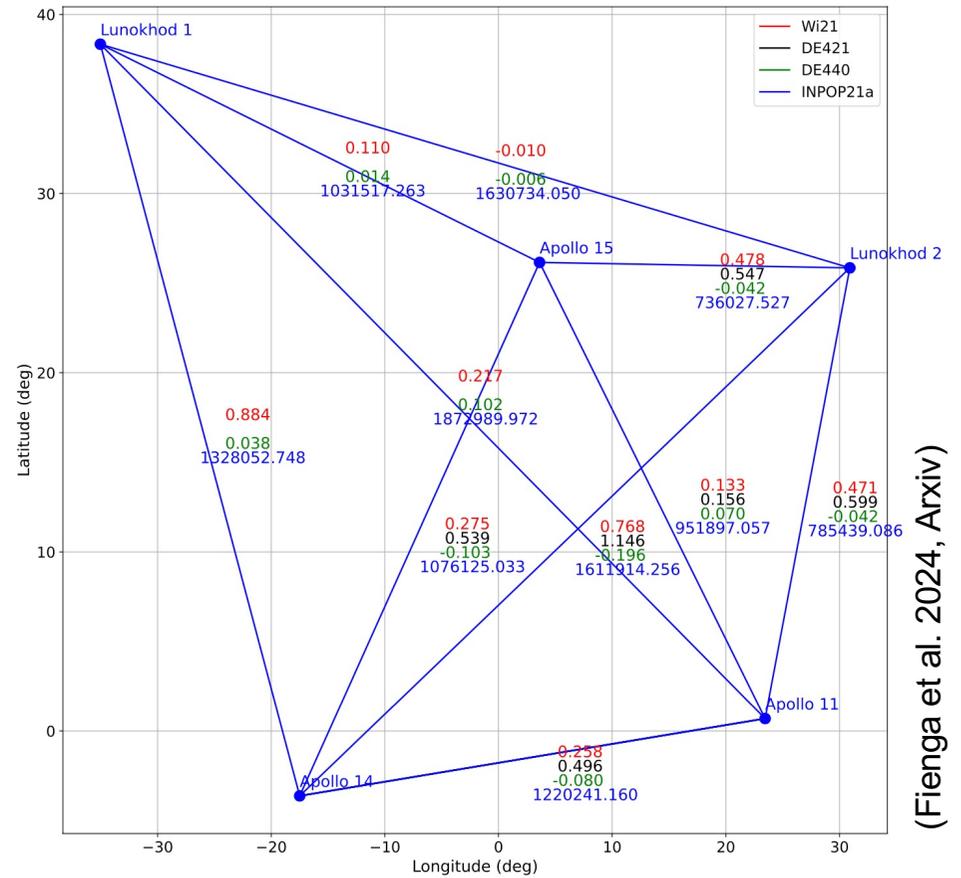
Ephemeris +
space missions
results

Mean Earth direction
(gravity field, dissipation)
LG CWG ME-DE421

4. Materialisation –reference frame



Retroreflector positions from Lunar laser-ranging



(Fienga et al. 2024, Arxiv)

Lunar Laser Ranging

Retroreflectors positions (in PA)

- Favored control points are the five LLRR
- Formal/internal uncertainties in LLRR position is **40 cm** for DE (IOM-DE) and about **10 cm** INPOP
- External accuracy is about **2 meters** for EPM vs DE, INPOP and **1 meter** for INPOP and DE440 (ELPM, Pavlov 2020; DE Folkner et al. 2014 Park et al. 2021; INPOP Fienga et al. 2021)

	A11	A14	A15	L1	L2
	[m]	[m]	[m]	[m]	[m]
I17a-I21a	0.108	0.101	0.148	0.249	0.186
EPM-I17a	2.3	2.2	2.2	2.2	1.9
I21a-DE421	1.361	1.601	1.409	-	1.156
I21a-DE440	0.898	0.838	1.030	0.837	0.949
I19a-I20a	0.078	0.051	0.091	0.083	0.061

Construction of reference system and frame

Steps	Specific name	PA	ME
1. Conception	Ideal reference system	Diagonal matrix inertia	Mean Earth direction
2. Choice of a structure	Reference system	Undistorted moon	Gravity field, dissipative moon (Moon interior, tides)
3. Structure modelling	Conventional reference system	GRAIL and lunar ephemeris (DE, EPM, INPOP)	GRAIL and lunar ephemeris (DE, EPM, INPOP)
4. Materialisation	Conventional reference frame	LLR or future geodetic coordinates	LLR or future geodetic coordinates
5. Extension and densification	Secondary reference frame	Using new devices (e.g. VLBI) or craters position	Using new devices (e.g. VLBI) or craters position

(Rambaux et al., A&A, 2026)

Link with LRO frame

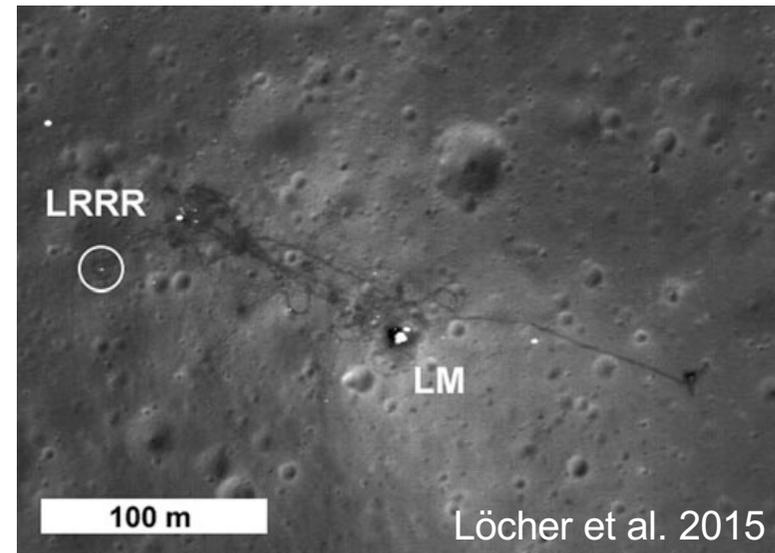
- LRO pictures have been taken of LLRR @ Moon surface
- LRO altimetric measurements @ LLRR

	Original LOLA frame	New LOLA frame	All available
Distance	[m]	[m]	[m]
Apollo 11	9.10	7.53	6.65
Apollo 14	7.83	7.15	6.63
Lunokhod 2	7.90	9.92	11.27

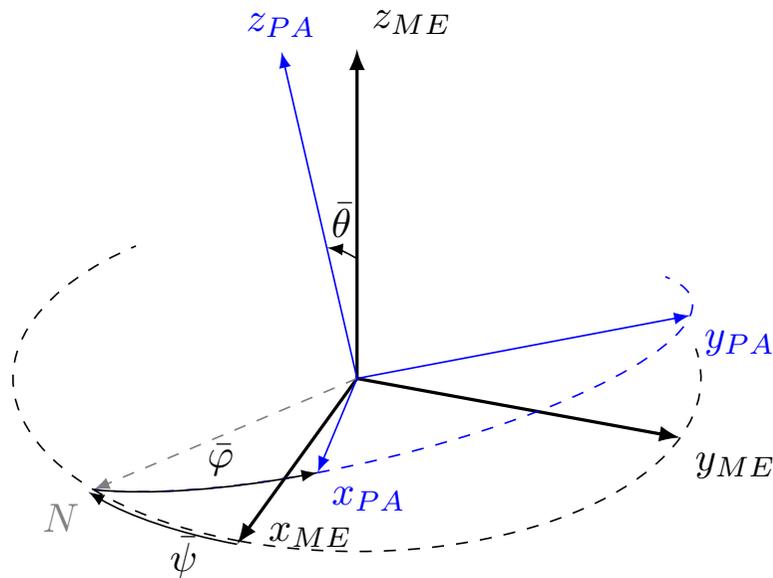
Table 3 from (Gläser et al 2018).

The horizontal distances in meters between the LRRR in the NAC DTM and the observed coordinates through lunar laser ranging (DE421).

- Tie to LRO DTM at **5-10 meters** accuracy, due to LRO orbit determination accuracy (Wagner et al. 2024)



Transformation ME - PA



Approximate transformation
(e.g. Park et al. 2021)

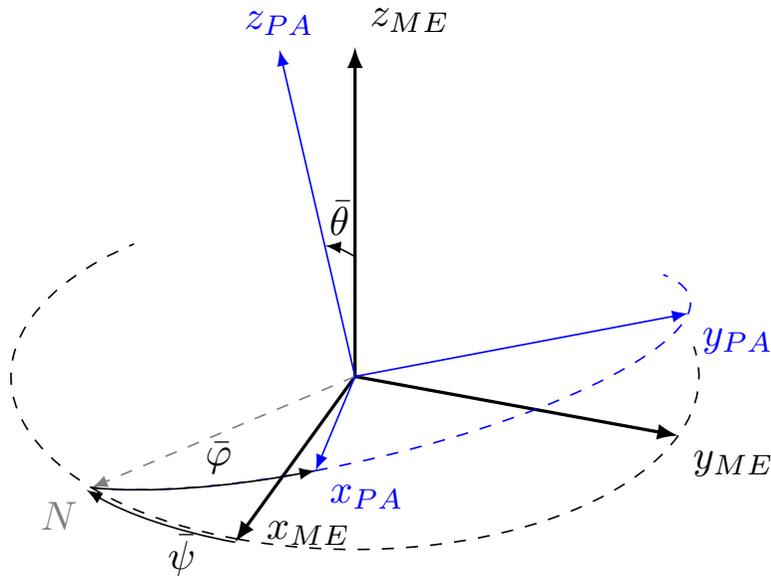
$$X_{\mathcal{M}} = R_x(-\bar{p}_2)R_y(\bar{p}_1)R_z(-\bar{\tau} + I^2\frac{\bar{\sigma}}{2})X_{\mathcal{P}}$$

$$\bar{p}_1 = \sin \bar{\theta} \sin \bar{\varphi}$$

$$\bar{p}_2 = \sin \bar{\theta} \cos \bar{\varphi}$$

$$\bar{\tau} = \bar{\psi} + \bar{\varphi}$$

Transformation ME - PA



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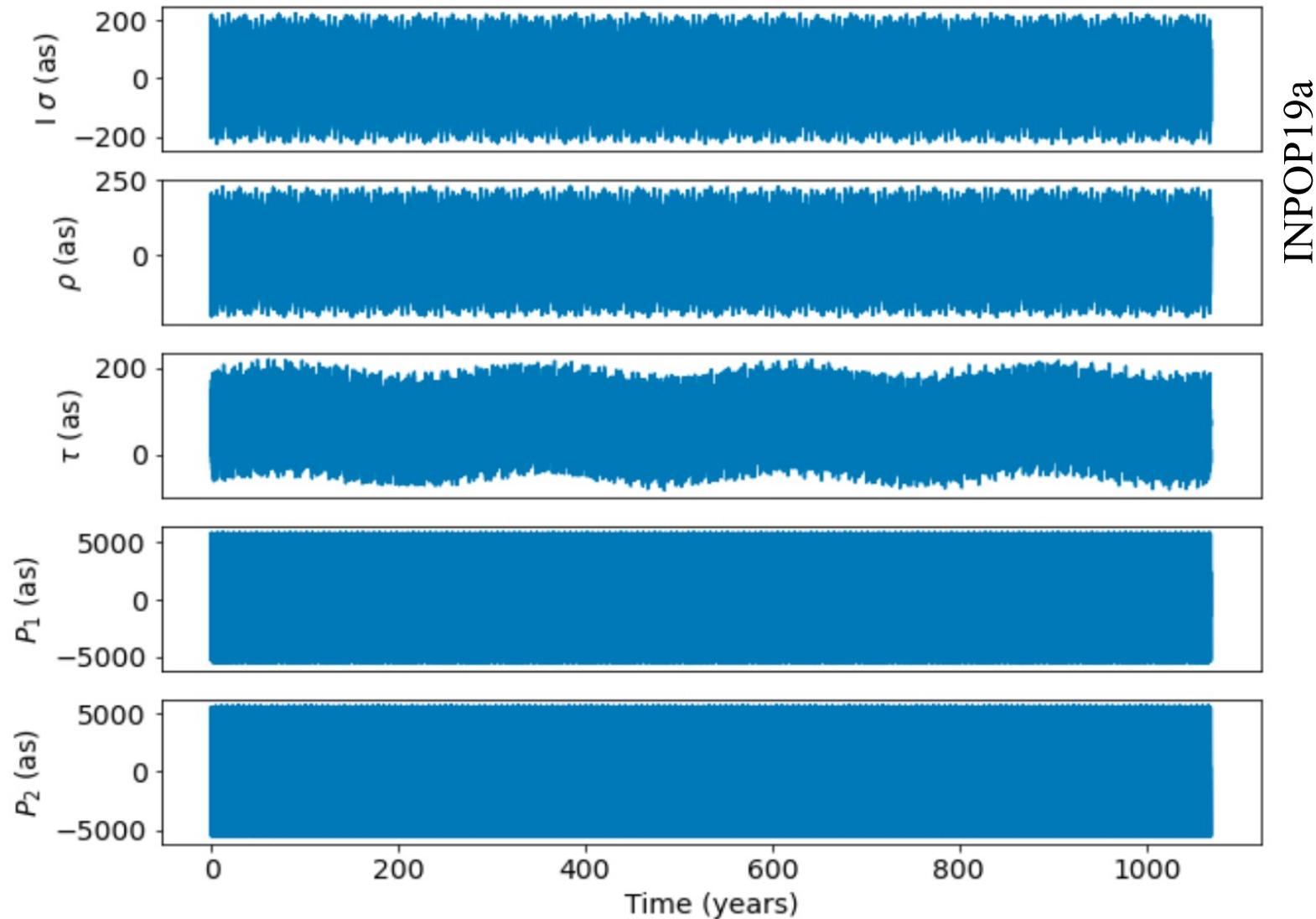
Full transformation
(Rambaux et al. 2026)

$$R_z(\bar{\phi})R_x(\bar{\theta})R_z(\bar{\psi})X_{\mathcal{M}} = X_{\mathcal{P}}$$

Difference in positions for all
LRR is about 20 cm.

Extraction of constant terms

1 as \leftrightarrow 8 m



Procedure to determine constant terms

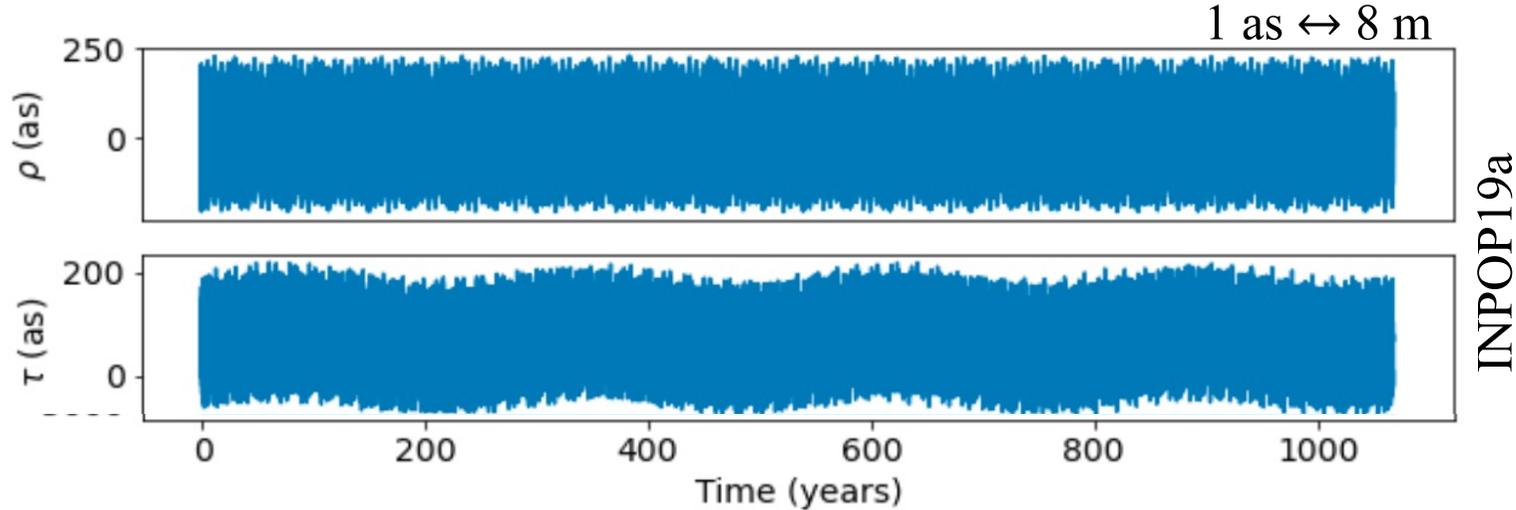
(Rambaux and Williams 2011)

1. Fit of the ephemeris (**DE421, DE430, INPOP19a**) of Moon rotation (**1070** years) with the following test functions (Polynomial, Fourier, Poisson):

$$f(t) = \sum_j^m a_j t^j + \sum_i^n (C_i + \epsilon_i^c t) \cos(\phi_i(t)) + (S_i + \epsilon_i^s t) \sin(\phi_i(t))$$

2. The frequencies are identified by a **frequency analysis** (Laskar 1998, 2005) and compared to analytical and semi-analytical theories (Eckhardt 1981, Chapront et al. 1999) or previous analysis (Newhall and Williams 1997, Rambaux and Williams 2011)
3. Then **least-square reduction method** required to take into account varying frequencies and amplitudes in the fit.
4. The combination of the two methods is iterative.

Extraction of constant terms



	Constant (as)	No. of frequencies	RMS (mas)
$I\bar{\sigma}$	-0.247	180	6.9
$\bar{\rho}$	-0.011	162	8.4
$\bar{\tau}$	67.589	169	14.2
p_1	-78.595	182	3.9
p_2	+0.290	151	4.4

	\bar{p}_1	\bar{p}_2	$\bar{\tau} - I^2 \frac{\bar{\sigma}}{2}$	$I\bar{\sigma}$	$\bar{\tau}$
DE421 ^(b)	-78.513	+0.290	+67.750	-0.249	67.753
DE421 IOM ^(a,b)	-78.513	+0.290	+67.756		
DE430 IOM	-78.580	+0.285	+67.573		
DE440 ^(c)	-78.6944	+0.2785	+67.8529		
DE440 IOM	-78.694	+0.278	+67.853		
INPOP19a ^(d)	-78.5954	+0.2903	+67.5881	-0.2471	67.5892

(a) Williams & Boggs (2008b), (b) Rambaux & Williams (2011)

(c) Park et al. (2021), (d) Rambaux et al. 2026

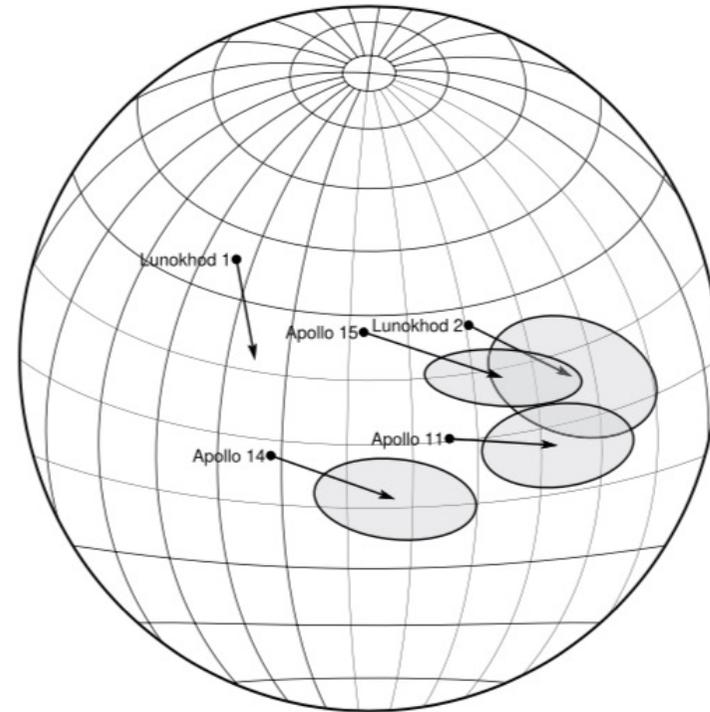
Open questions

→ **Empirical terms** introduced in order to fit the LLR data at <cm level

→ **Stability of LLR positions**

The weighted average rate of four LLR is 8.3 ± 3.1 mm/year (Williams and Boggs 2021)

So it reflects a global motion of 40 cm in 50 years.



(Williams and Boggs 2021)

Conclusion

- The Moon is in spin-orbit resonance and its rotation follows Cassini laws
- The Lunar reference system (LRS) for the Moon is described by the PA and the ME.
- These LRS require the gravity field and ephemeris. The PA can be improved by multi-geodetic techniques (e.g. Argonaut/Novamoon ESA).
- The internal accuracy is about 10's cm and external accuracy about 1 m.
- Description of the proposition for International Lunar Reference Frame (ILuRF) next talk.



Rambaux et al. 2026, A&A
Lunar reference systems and their realisations
using INPOP ephemerides

