Lunar Data Network PNT Systems Capability and Interfaces Status

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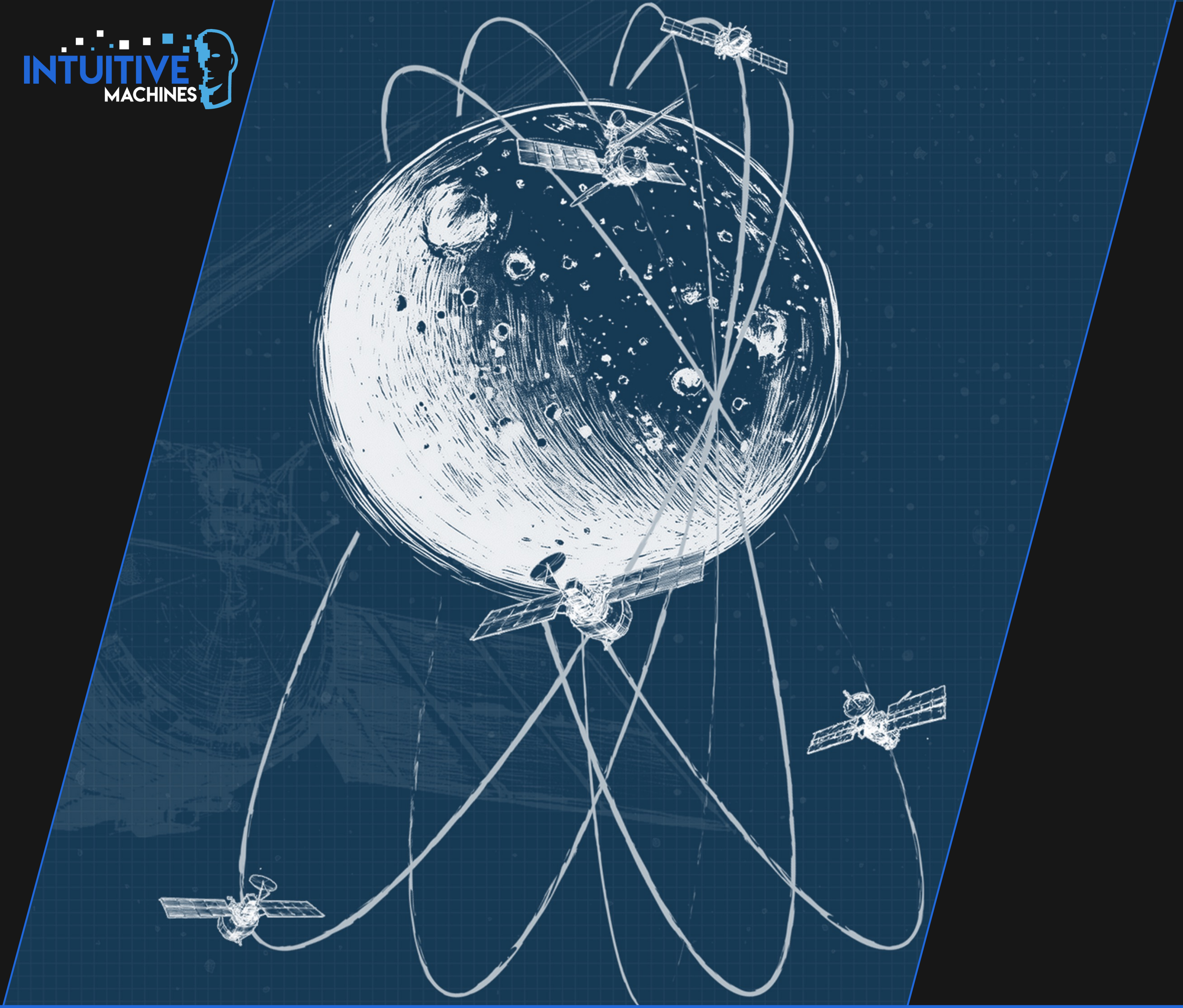
PNT System Design Updates

- Minor revisions to ELFO constellation
- AFS Message generation and parameterization
  - Ephemeris
  - Clock
  - Frame Rotations
- AFS Message bit allocations
- P2P ConOps development and user interface with data formatting
- Relativistic Time Considerations and broadcasting a consistent Lunar relative ephemeris and time
- Drafts of Lunar User Guide (LUG) and related ICDs for user implementation of algorithms



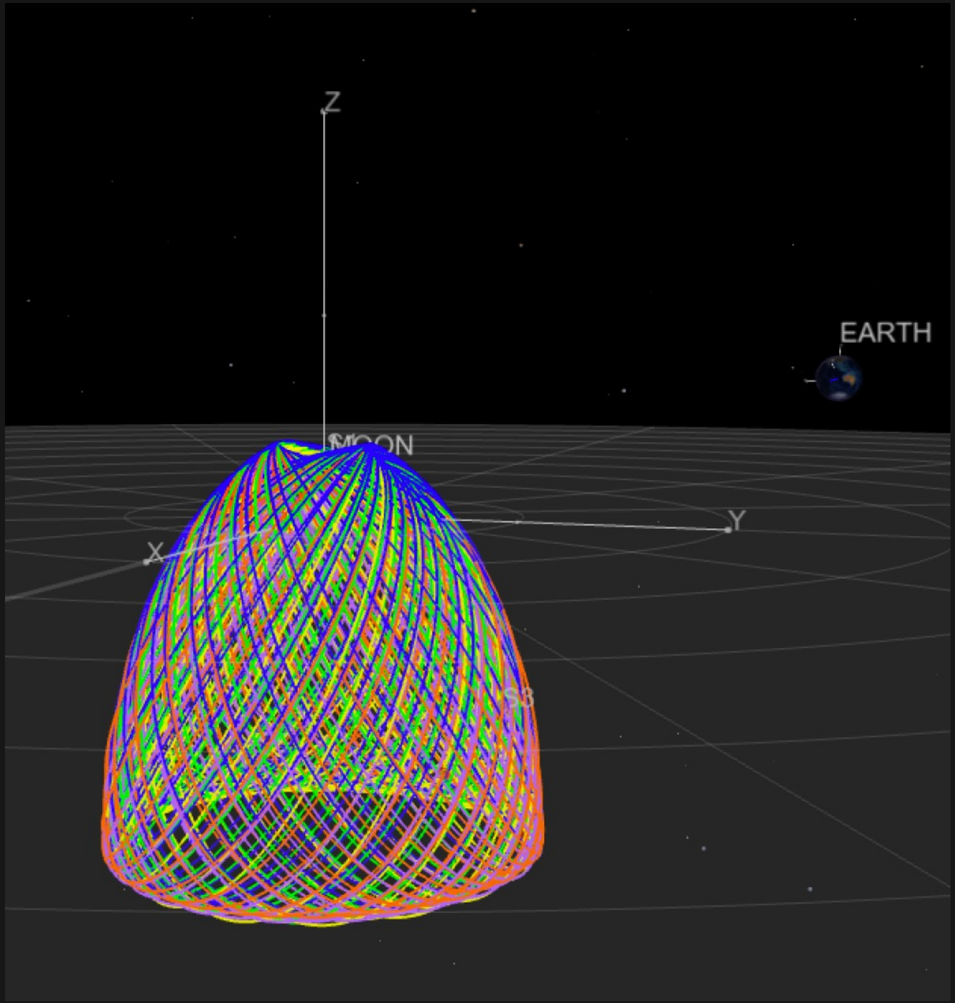
Current ELFO Constellation Design

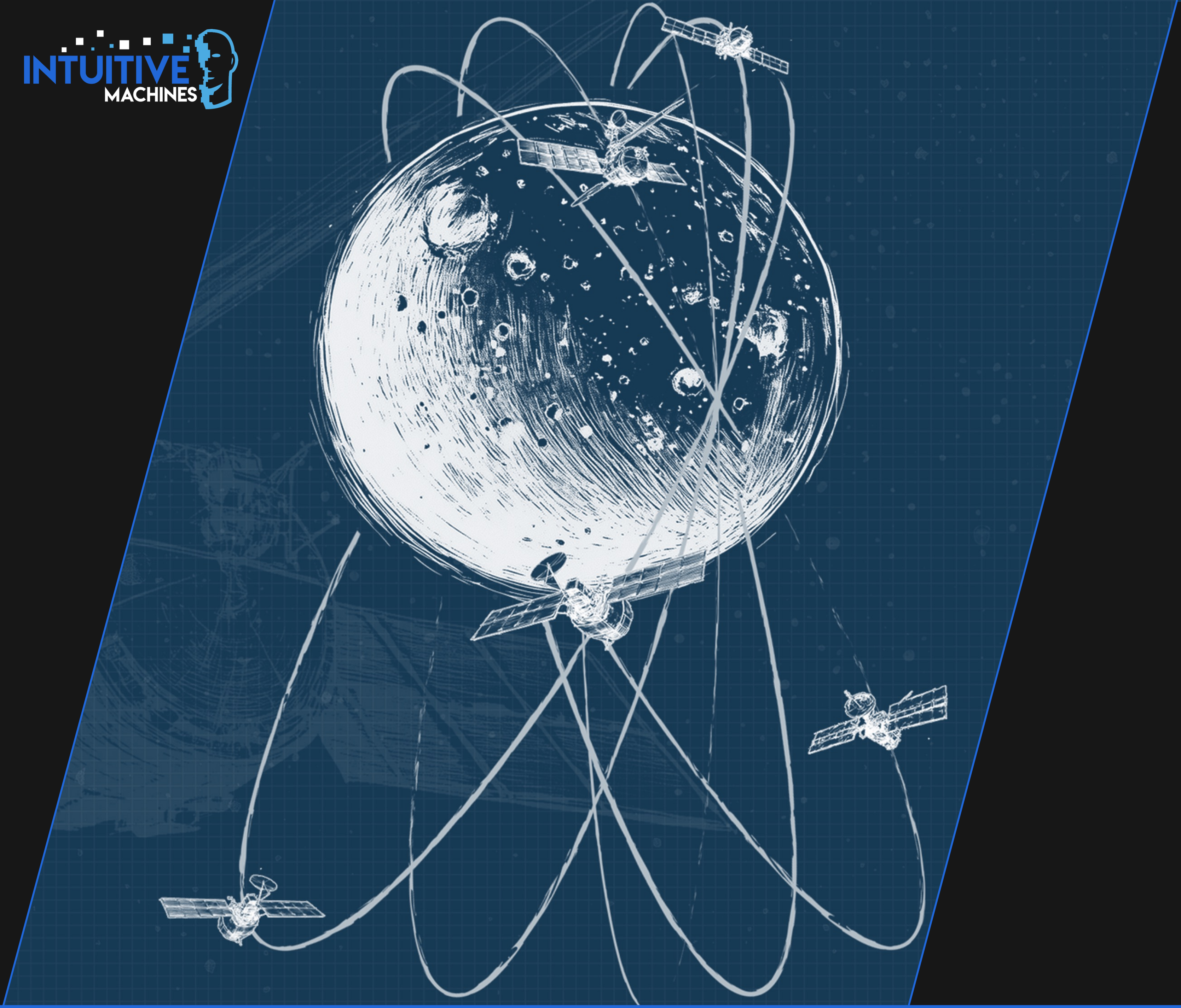
Constellation Modeling

ELFO Constellation Design

- 5-satellite ELFO constellation optimized for South Pole coverage and LCRNS GDOP requirements.
- Representative orbit: a ≈ 12,000 km, e ≈ 0.68, i ≈ 56°, orbital period ≈ 32.8 hr.
- Distributed across three planes, including two planes with ~180° mean-anomaly separation and a single-sat plane.
- Trajectory differences yield small satellite-to-satellite differences in mean drift and periodic amplitude.

	$\bar{a}$ [km]	$\bar{a}$ [°]	$\bar{e}$	$\bar{h}_p$ [km]	$\bar{h}_a$ [km]
LDN-1	12001.5	57.48	0.681	2091.2	18437.1
LDN-2	12002.4	56.54	0.684	2058.2	18471.9
LDN-3	11991.0	57.42	0.688	2001.0	18506.1
LDN-4	12007.6	56.08	0.691	1971.4	18569.0
LDN-5	12006.1	56.23	0.689	2000.6	18536.7





AFS Message and LUG Status

AFS Message Status Summary

Fundamental Messages for LNIS v5

- IM conducting analysis of the LNIS fundamental message trades for coordination with the LNIS PNT WG
- Draft Lunar User Guide (LUG) and ICD outlining algorithms and AFS data structure near completion

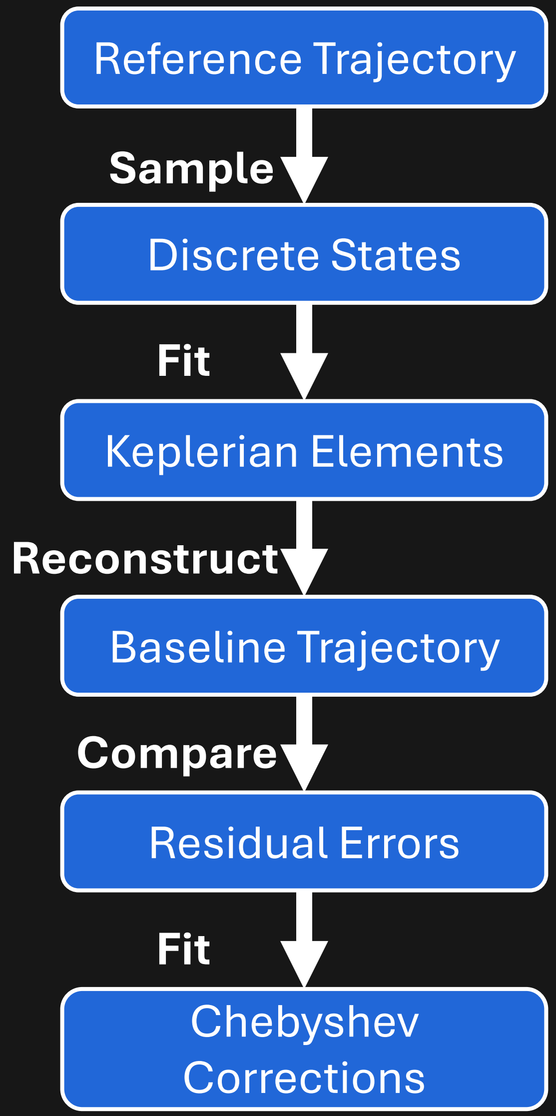
MSG	Subframe	MSG Title	Progress Summary
G1	SB4	LunaNet Network Access Information	In Progress
G2	SB2	Health and Safety	In Progress
G4	SB2	Clock and Ephemeris (CED) Data	Near Complete
G5	SB3 or SB4	Multiple Orbit Almanac	In Progress
G8	SB1, SB2, & SB3	Time of Transmission (ToT)	Defined
G10	SB3 or SB4	Maneuver	In Progress
G30	SB2	Time Conversions	Near Complete
G32	SB3 or SB4	Coordinate Frame Conversions	Near Complete

Clock & Ephemeris Data

LDN Algorithm Overview
Keplerian + Chebyshev Correction

LDN Representation Fitting Overview

1. Sample the reference trajectory in Moon-centered Inertial frame
2. Fit Keplerian elements via least squares
3. Reconstruct state from Keplerian elements
   - Moon PA, apply necessary rotations
4. Compute position residual errors
   - Compute RIC frame using solved Keplerian representation and apply to residuals
5. Fit Chebyshev polynomials to residuals
   - Compute the required scale factor for the coefficients
   - Chebyshev derivatives used in computing velocity representation



Coordinate Frame Transformation

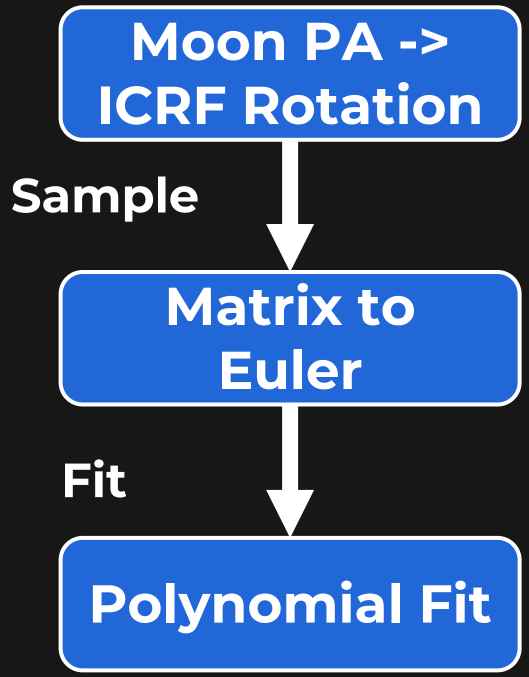
User Algorithms

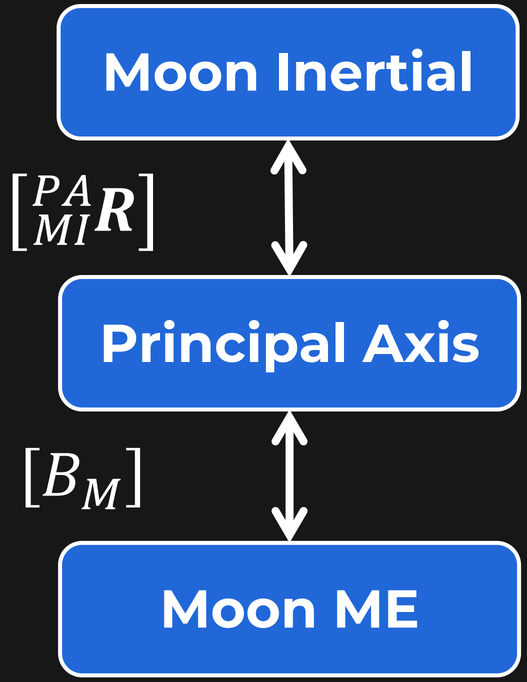
Rotation Matrices

- Derivative of rotation matrices used in rotating the velocity
- User algorithm utilizes 3-1-3 Euler rotations

Position and Velocity

- Use the drift and drift rate terms to compute the Euler angle rates to calculate the derivative of the R1 and R3 rotations
- May use this method to solve position and velocity in Moon ICRF, Moon PA, or Moon ME





Sample User Algorithms (LUG)

User Algorithms
(Keplerian -> Moon PA)

Perifocal Pos/Vel User Algorithm

Table 5 – Navigation User Equations (Perifocal Position/Velocity)

Element/Equation	Description
$\mu = 4902.800118 \frac{km^3}{sec^2}$	JPL planetary ephemeris DE440 value of Moon's gravitational constant
$n_0 = \sqrt{\frac{\mu}{A^3}}$	Computed Mean Motion (rad/sec)
$t_k = t - t_{oe}$	Time from Ephemeris Reference Time
$M_k = M_0 + n_0 t_k$	Mean Anomaly
	Kepler's equation ($M_k = E_k - e \sin E_k$) may be solved for Eccentric Anomaly (E_k) by iteration:
$E_0 = M_k$	- Initial Value (radians)
$E_j = E_{j-1} + \frac{M_k - E_{j-1} + e \sin E_{j-1}}{1 - e \cos E_{j-1}}$	- Refined Value
$E_k = E_j$	- Final Value (radians)
$v_k = \arctan2\left(\sin(E_k)\sqrt{1-e^2}, \cos(E) - e\right)$	True Anomaly (unambiguous quadrant)
$r_k = A(1 - e \cos E)$	Radius (km)
$\left.\begin{array}{l} x_{pf} = r_k \cos v \\ y_{pf} = r_k \sin v \\ z_{pf} = 0 \end{array}\right\}$	Perifocal Position (km)
$\left.\begin{array}{l} \dot{x}_{pf} = -\sin E \frac{\sqrt{\mu A}}{r_k} \\ \dot{y}_{pf} = \cos E \sqrt{1-e^2} \frac{\sqrt{\mu A}}{r_k} \\ \dot{z}_{pf} = 0 \end{array}\right\}$	Perifocal Velocity (km/s)

Rotation User Algorithm

Table 6 – Rotation from Perifocal to Moon PA

Element/Equation	Description
$\dot{\Omega}_M = 2.6616994576329732E-06 \frac{rad}{s}$	Rotation rate of the Moon
$\vec{\dot{\Omega}}_M = [0 \quad 0 \quad \dot{\Omega}_M]$	Define the rotation rate vector
$R_{3,\Omega} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$	R_3 Rotation with right ascension
$R_{1,i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix}$	R_1 Rotation with inclination
$R_{3,\omega} = \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$	R_3 Rotation with argument of periapsis
$Q = R_{3,\Omega} R_{1,i} R_{3,\omega}$	3-1-3 Rotation from Perifocal to Quasi-Inertial frame
$\left.\begin{array}{l} r_{pf} = [x_{pf} \quad y_{pf} \quad z_{pf}]^T \\ v_{pf} = [\dot{x}_{pf} \quad \dot{y}_{pf} \quad \dot{z}_{pf}]^T \end{array}\right\}$	Define perifocal position and velocity vectors
$\left.\begin{array}{l} r_q = Q\, r_{pf} \\ v_q = Q\, v_{pf} \end{array}\right\}$	Rotate to Quasi-Inertial frame
$\left.\begin{array}{l} r_q = [x_k \quad y_k \quad z_k]^T \\ v_q = [\dot{x}_k \quad \dot{y}_k \quad \dot{z}_k]^T \end{array}\right\}$	Define Quasi-Inertial position and velocity vectors
$\phi = \dot{\Omega}_M (t - t_{oe})$	Pole rotation since epoch
$R_{Q \to PA} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$	Rotation to align z-axis of Quasi-Inertial frame to Moon PA
$\left.\begin{array}{l} r_{PA} = R_{Q \to PA} r_q \\ v_{PA} = R_{Q \to PA}\, v_q - \vec{\dot{\Omega}}_M \times r_{PA} \end{array}\right\}$	Rotate Quasi-Inertial position and velocity to Moon PA position and velocity

Coordinate Frame Transformation

Coordinate Frame Conversions Data Fields

Data Fields Binary Representation

Bit Allocation

- Coordinate Frame Conversions (MSG-G32) is allocated to Subframe 3
- The Euler rotation fitting algorithm consists of 3 sets of quadratic polynomials which produce 3 coefficients each:
  - Bias
  - Drift
  - Drift Rate
  - Total Bits = **312 bits**
- This bit allocation is conservative and is likely to be further reduced
- Possible opportunities to quantize values using a similar approach to CED
  - Range-based quantization is effective for bias values
  - Common scale factor quantization is effective for drift and drift rate terms

Table 9 – Coordinate Frame Conversions Fields Binary Representation

Parameter	Definition	Bits (TBR)	Scale Factor	Unit	Values
ϕ_0	ϕ Bias	40	--	rad	$0...2\pi$
ϕ_1	ϕ Drift	32	TBD	$\frac{rad}{s}$	N/A
ϕ_2	ϕ Drift Rate	32	TBD	$\frac{rad}{s^2}$	N/A
θ_0	θ Bias	40	--	rad	$0...2\pi$
θ_1	θ Drift	32	TBD	$\frac{rad}{s}$	N/A
θ_2	θ Drift Rate	32	TBD	$\frac{rad}{s^2}$	N/A
ψ_0	ψ Bias	40	--	rad	$0...2\pi$
ψ_1	ψ Drift	32	TBD	$\frac{rad}{s}$	N/A
ψ_2	ψ Drift Rate	32	TBD	$\frac{rad}{s^2}$	N/A

Clock & Ephemeris Data

Forward Work

URA

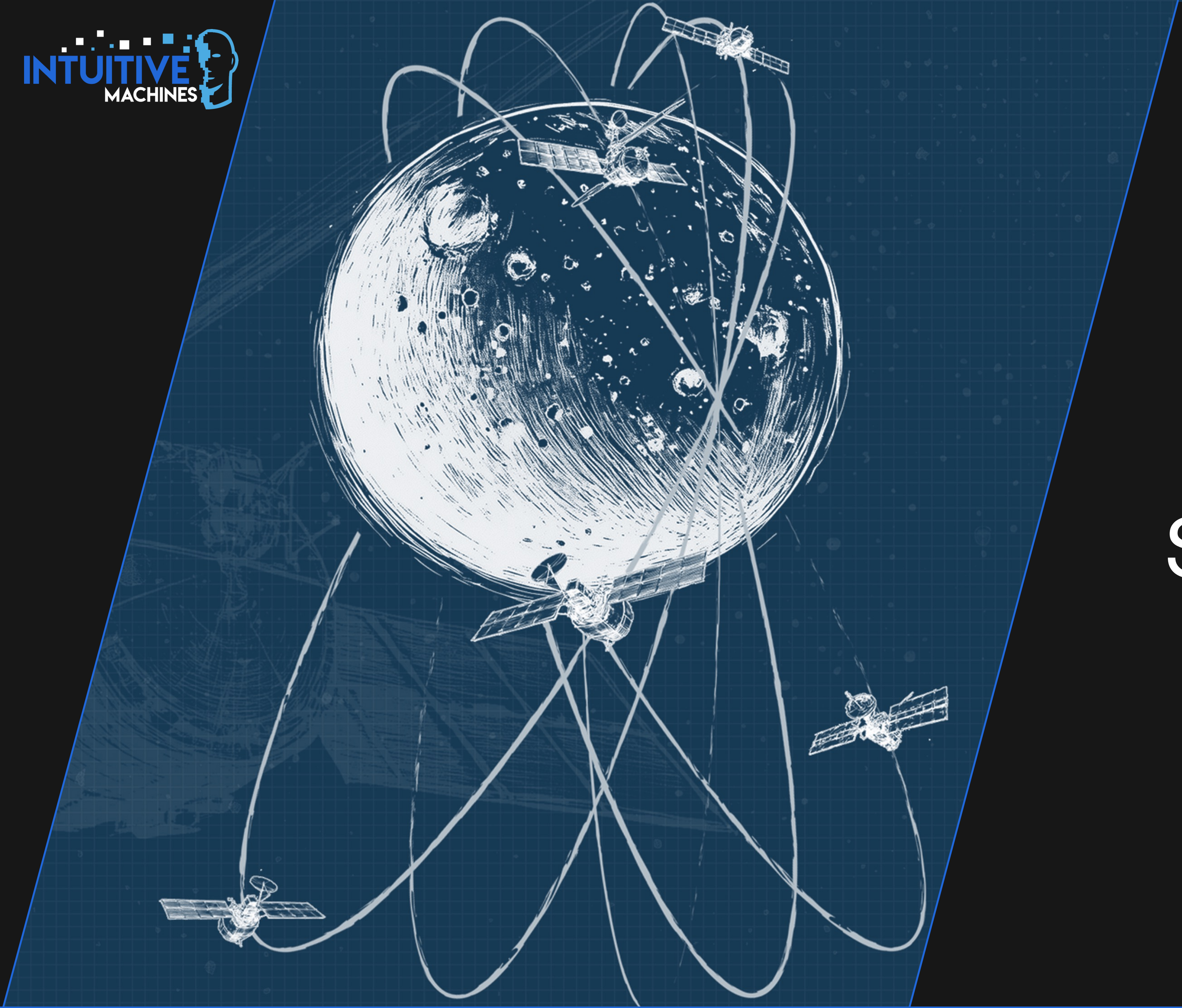
- The expected quality estimated by the LNSP of the data will be included in the message
- As GPS transmits URA or Galileo SISA
- Current strategy is to put a URA-like parameter in CED (5-bits) as the core accuracy measurement and add higher accuracy SISA-like parameter to ancillary information (SB3 or SB4)

Clock Corrections

- Clock parameters to be included in the CED message:
  - Time of Clock
  - Clock Bias
  - Clock Drift
  - Clock Drift Rate
- Currently reserving 120 bits for these parameters
- Final bit allocation needs to be decided

Bit Allocations/Other

- IM currently has a conservative estimate of bit allocation.
- Improvements may be realized in the following areas with improved quantization:
  - Reference time
  - Chebyshev coefficients
  - Validity period
  - Keplerian elements
- Continue analysis with phase center correction applied and potential degraded performance during slews



Point-to-Point Service Status

Point-to-Point Services

P2P PNT ConOps

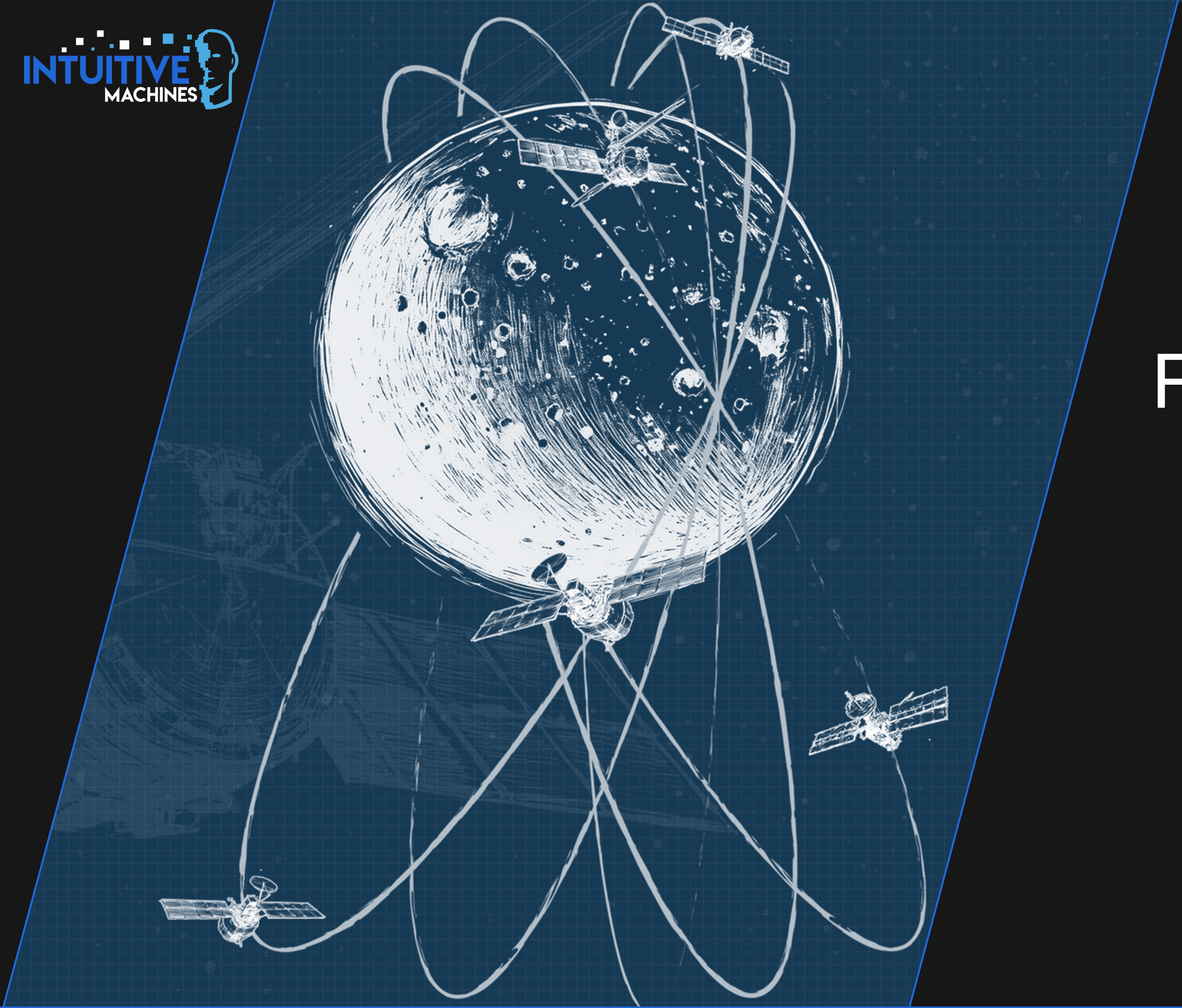
Current Status

- Preliminary workflow of user process
  - On-boarding, scheduling, through data delivery
- Initial definition of P2P data structure

Forward Work

- Resolve preferred method for ephemeris delivery for P2P data dissemination
- Finalize P2P tracking data format standard
- Develop robust asset scheduling

PNT Service ID	Services	Band		S-Band				Ka-Band
		Modulation		BPSK	PCM/PM/bi-phase-L w. PN ranging	PCM/PSK/PM NRZ-L w. PN ranging	AFS (Augmented Forward Signal)	BPSK
		Ranging		No	PN CCSDS 414.1-B-2	PN CCSDS 414.1-B-2	CDMA PN	Potential for telemetry ranging
		Symbol Rates	Min	2 ksps	48 ksps	0.5 ksps	250 sps	1 Msps
			Max	2 Msps	1.024 Msps	48 ksps	1 ksps	2.048 Msps
		Interop Signal ID		PFS1a	PFS1b	PFS1c	PFS5	PFKa1
1wDRef	1-Way FWD Doppler Reference			X	X	X	X	X
1wRTRef	FWD Pseduo-Range and Timing Ref						X	
2wDMeas	2-Way Doppler Measurement			X	X	X		
2wRMeas	2-Way Range Measurement				X	X		



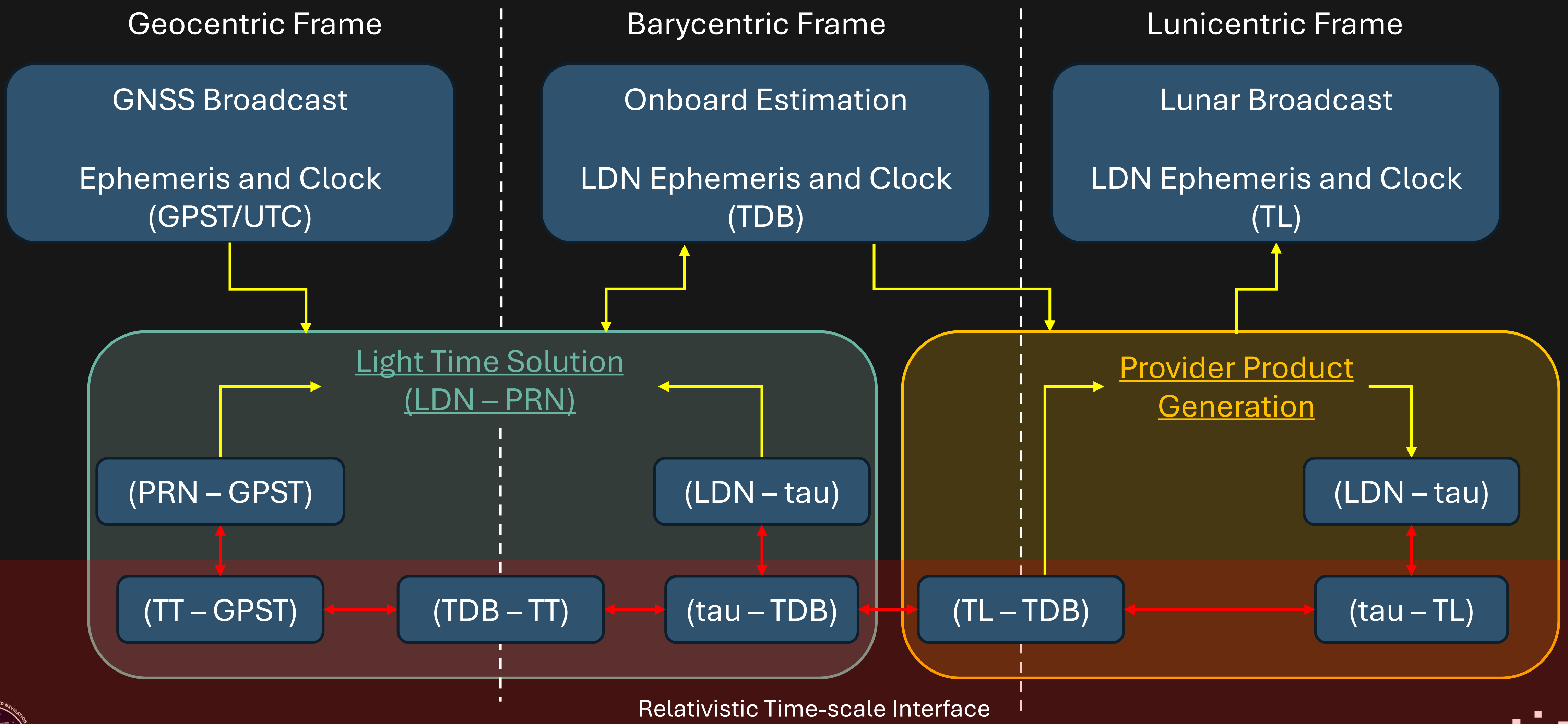
Relativistic Time Dissemination through AFS Broadcast

Current IM Approach for Disseminating Lunar Time

- IM is currently developing our LDN satellites to provide PNT services to the Moon
  - Compliant with LCRNS SRD and LunaNet LNIS requirements
  - 5 satellite ELFO design focused on providing service to the South Pole of the Moon
- LDN must provide a consistent lunar time product to Lunar users
  - Currently, the community has yet to agree on an international standard for what these time systems are
  - ELFO proper time rate varies throughout the eccentric orbit
  - GNSS as a primary data source means high-precision time transformation is necessary
- IM has adopted the following approach
  - Internal computation is done in TDB/BCRS
    - Orbit design
    - Estimation of ephemeris and clock
    - Light-time solutions to GNSS
  - Outward facing service is provided in TL/LCRS
    - Conversion from TDB to TL/TCL (or other)
    - Software layer that can be updated as the international standards are defined

Relativistic Reference Systems Used

Time Scales and their role in operations



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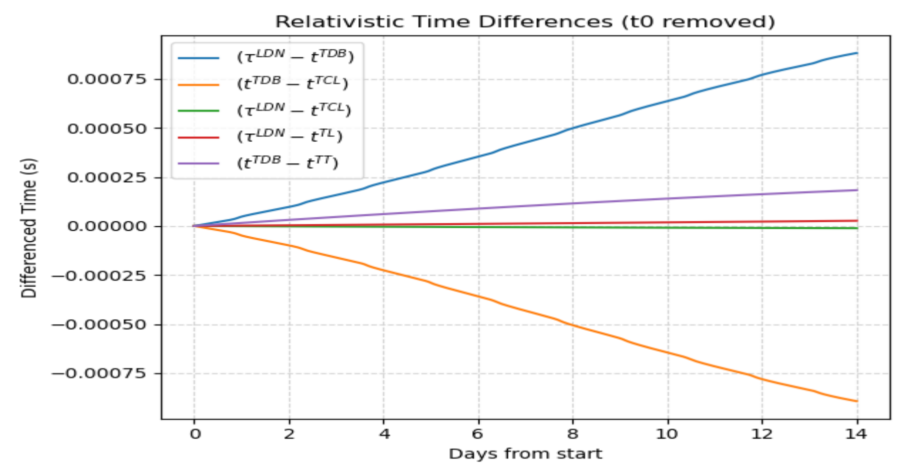
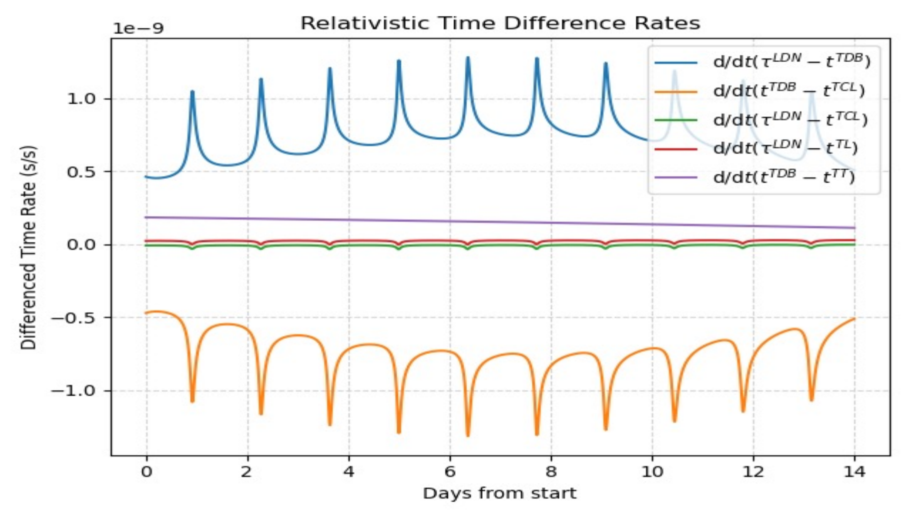
Relativistic Reference Systems Used

- Integration of the time system differences show the expected secular and periodic structures we'd expect.
- Differencing the LDN proper time relative to TDB with the known TDB – TCL removes most of the known average drift of the LCRS
- The remaining secular drift and periodic terms of the LDN proper time relative to TCL or TL remain small
- Periodic drift rates increase near the ELFO lunar periapsis passage.

Proper Time realizations near the Moon





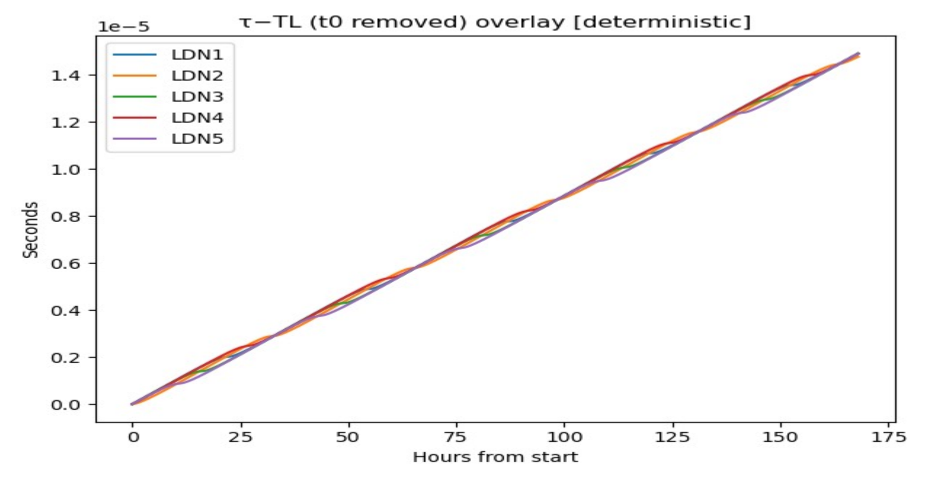
ELFO Spacecraft Clock Modeling

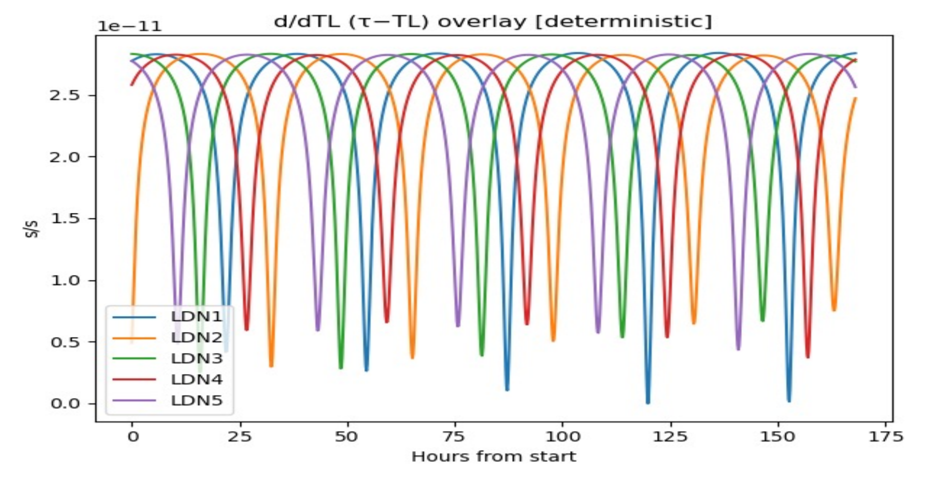
- Based on the ELFO orbit, we should expect a drift-rate relative to TCL as
- Average time drift rates were computed over 1-year ELFO simulation span
- Relativistic dot product term produces peak-to-peak 120 ns

$$\Delta t_{\mathrm{rel}}(t) = -\frac{2}{c^2}\mathbf{r}^{\mathrm{TL}}(t) \cdot \mathbf{v}^{\mathrm{TL}}(t).$$

	$\langle \tau^{LDN} - t^{TDB} \rangle$ [μs/day]	$\langle \tau^{LDN} - t^{TCL} \rangle$ [μs/day]	$\langle \tau^{LDN} - t^{TL} \rangle$ [μs/day]
LDN-1	58.732905	-0.590033	2.122110
LDN-2	58.724856	-0.589926	2.122217
LDN-3	58.732834	-0.590541	2.121602
LDN-4	58.742710	-0.589696	2.122447
LDN-5	58.719093	-0.589740	2.122403

ELFO Constellation Proper Time





ELFO Spacecraft Clock Modeling

Traceability to GNSS Time

- IOC will rely on GNSS signals for the primary synchronization of time
  - Our realization of TL is directly linked to GPST/UTC
  - It is only a forward one-way synchronization
    - Must account for deterministic relativistic models for proper estimation
  - On-board clock estimates can be chained to the GPST/UTC realization
- This architecture does not by itself produce a fully unified lunar time solution across the constellation
- In order to get a unified ensemble lunar time realization we need
  - inter-satellite crosslink time transfer so that clock offsets and drifts can be observed and estimated within the constellation,
  - two-way time and frequency transfer to reduce path-delay systematics and establish higher-quality traceability,
  - stable lunar reference clocks at known locations for long-term frequency accuracy.

Deriving Time from GNSS Data

Outputs and Connection to Broadcast Ephemeris

- With this, the onboard estimator produces:
  - A best-estimate spacecraft orbit state referenced to the internal TDB time argument, suitable for propagation and for consistent use in the light-time computation.
  - Residual receiver clock parameters (a_0, a_1, a_2) representing bias, drift, and drift rate relative to the internal time argument after deterministic relativistic mapping is applied.
  - Optional calibrated or estimated hardware delay parameters.
- Everything internal to the LDN satellite stays in the TDB time argument
- The next step is to make the user facing output product TL consistent
  - States will need to be converted
  - Times will need to be converted
  - The clock model will need to be converted
- The benefit is that this is just an application layer to the flight-software for outputting and encoding onto the AFS signal

Lunar Time Realization and Dissemination

LDN Broadcast Time Parameterization

- Plan is to broadcast an ephemeris and clock model relative to LDN Lunar Time (TL)
- Since we are estimating in TDB, we need to convert these states and times to a TL time

$$\mathbf{x}_T^{\mathrm{LCRS}}(t_T^{\mathrm{TL}}) = \mathcal{T}_{\mathrm{BCRS}\to\mathrm{LCRS}}\left(\mathbf{x}_T^{\mathrm{BCRS}}(t_T^{\mathrm{TDB}}), t_T^{\mathrm{TDB}}\right)$$

$$\epsilon^{\mathrm{TDB}}(t) = a_0 + a_1(t - t_0^{\mathrm{TDB}}) + \tfrac{1}{2}a_2(t - t_0^{\mathrm{TDB}})^2.$$

- States need to be scaled using a Lorentz transformation
  - Sample on a grid of TDB times and convert the ephemeris be consistent with TL
- The TDB time series for the states need to be converted to TL
  - Relativistic clock effect is removed from the time series using the TL states

$$\Delta t_{\mathrm{rel}}(t) = -\frac{2}{c^2}\mathbf{r}^{\mathrm{TL}}(t)\cdot\mathbf{v}^{\mathrm{TL}}(t).$$

- A TL consistent clock polynomial is fit to this new TL time series
- The TL consistent ephemeris and clock model are then converted to the broadcast format on the AFS signal

$$\delta t_{\mathrm{poly}}^{\mathrm{TL}}(t) = b_0 + b_1(t - t_0^{\mathrm{TL}}) + \tfrac{1}{2}b_2(t - t_0^{\mathrm{TL}})^2,$$

Lunar Time Realization and Dissemination

IM Provided TL broadcast from TDB

- We analyze a sliding window fit of this algorithm with broadcasts ranging from 1 hours to 24 hours
  - We anticipate updating the ephemeris at more frequent than every 2 hours
- We simulated random clock predictions in addition to the proper time deviation
- We compare the accuracy of the TL based fit to the true TL time series
- In general, this method can convey the appropriate time in TL to sub ns over a 24 hour fit
  - Fitting every hour to two hours is ps level